ELECTROSTATICS

- Coulomb's law : $F = \frac{k\,q_1q_2}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$
- Relative permittivity or dielectric constant :

$$\varepsilon_r \text{ or } K = \frac{\varepsilon}{\varepsilon_0}$$

- Linear charge density : $\lambda = \frac{\text{charge}}{\text{length}}$
- Surface charge density : $\sigma = \frac{\text{charge}}{\text{area}}$
- Volume charge density : $\rho = \frac{\text{charge}}{\text{volume}}$
- Electric field intensity at a point at a distant r from a point charge q is $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$
- Electric dipole moment, $|\vec{p}| = q2a$
- Electric field intensity on axial line (end on position) of the electric dipole :
  - At *a* point r from the centre of the electric dipole, E = $\frac{1}{4\pi\varepsilon_0}\frac{2pr}{(r^2-a^2)^2}$
  - At very large distance *i.e.*, ($r >> a$),

$$E = \frac{2p}{4\pi\varepsilon_0 r^3}$$

- Electric field intensity on equatorial line (board on position) of electric dipole
  - At the point at a distance r from the centre of electric dipole,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{(r^2+a^2)^{3/2}}$$

  - At very large distance *i.e.* $r >> a$,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$$

- Electric field intensity at any point due to an electric dipole, E = $\frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\sqrt{1+3\cos^2\theta}$
- Electric field intensity due to a charged ring
  - At a point on its axis at a distance r from its centre, E = $\frac{1}{4\pi\varepsilon_0}\frac{qr}{(r^2+a^2)^{3/2}}$
  - At very large distance i.e. r >> a,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

- Torque on an electric dipole placed in a uniform electric field, $\vec{\tau} = \vec{p}\times\vec{E}$ or $\tau = pE\sin\theta$
- Potential energy of an electric dipole in a uniform electric field is

$$u = pE(\cos\theta_2 - \cos\theta_1)$$

where θ_1 and θ_2 are initial angle and final angle between $\vec{p}$ and $\vec{E}$.

- Electric flux, $\phi = \vec{E}\cdot d\vec{S}$
- Gauss's law : $\oint = \vec{E}\cdot d\vec{S} = \frac{q}{\varepsilon_0}$
- Electric field due to a uniformly charged thin spherical shell of uniform surface charge density σ and radius R at a point distance r from the centre of the shell is givedn as follows :
  - At a point outside the shell *i.e.*, $r > R$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

  - At a point on the shell *i.e.*, $r = R$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$$

  - At point inside the shell *i.e.*, $r < R$

$$E = 0$$

where $q = 4\pi R^2\sigma$

- Electric field due to a rion conducting solid sphere of un orm volume charge density p and radius R at a point distant r form the centre of the sphere is given as follows :

- At a point outside the sphere *i.e.*, $r > R$

$$\text{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

- At a point outside the surface of the sphere *i.e.*, $r = R$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$$

- At a point *inside the sphere i.e.*, $r < R$

$$E = \frac{\rho r}{3\varepsilon_0} = \frac{1}{4\pi\varepsilon_0}\frac{qr}{R_3}$$

Where $q = \frac{4}{3}\pi R^3 \rho$

- Electric feld due to a thin non conducting infintite sheet of charge with uniform surface charge denity σ is

$$E = \frac{\sigma}{2\varepsilon_0}$$

- Electric feld between two infinite thin plane parallel sheets of uniform surface charge density σ and $-\sigma$ is

$$E = \frac{\sigma}{\varepsilon_0}$$

- Electric potential, $V = \frac{W}{q}$
- Electric potential at a point distance *r* from a point charge *q* is

$$V = \frac{q}{4\pi\varepsilon_0 r}$$

- The electric potential at a point due an electic dipole

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

  - When the point lies on the axial line of dipole *i.e.*, $r\ \theta = 0^{o}$.

$$V = \frac{p}{4\pi_0 r^2}$$

  - When the point on the equatorial line of the dipole, *i.e.*, $\theta = 90^{o}$.

$$\text{V} = 0.$$

- Electric potential due to a uniformly charged spherical shell of uniform surface surface charge density σ and radius R at a distance *r* from the centre of the shell is given as follows :
  - At a point outside the shell *i.e.*, $r > R$

$$\text{V} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

  - At a point on the shell *i.e.*, $r = R$

$$\text{V} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

  - At a point inside the shell *i.e.*, $r < R$

$$\text{V} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

- Electric potential due to a non- conducting solid sphere of uniform volume charge senity p and radius R at a sistance r from the sphere is given as follows
  - At a point outside the sphere *i.e.*, $r > R$

$$\text{V} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

  - At a point on the sphere *i.e.*, $r = R$

$$\text{V} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

  - At a point inside the sphere *i.e.*, $r < R$

$$\text{V} = \frac{1}{4\pi\varepsilon_0}\frac{q(3R^2 - r^2)}{2R^3}$$

- Relationship between $\vec{E}$ and $\vec{V}$

$$\vec{E} = -\vec{\nabla}V$$

where $\vec{\nabla} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]$

  - Negative sign shows that the direction of $\vec{E}$ is the direction of detential.

- Electric potential energy of a system of two point charges is

$$u = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

- Electric feld at the surface of a charged conductor

$$\vec{E} = \frac{\sigma}{\varepsilon} n$$

- Capacitance, $C = \frac{Q}{V}$
- Capacitance of a spherical conductor of radius R is

$$C = 4\pi\varepsilon_0 R$$

- Capacitance of an air filled parallel plate capacitor

$$C = \frac{\varepsilon_0 A}{d}$$

- Capacitance of an air filled spherical capacitor

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a}$$

- Capacitance of an air filled spherical capacitor

$$C = \frac{2\pi\varepsilon_0 L}{\text{In}\left(\frac{b}{a}\right)}$$

- Capacitance of a parallel plate capacitor with a dielectric slab of dielectric constant K, completely filled between the capacitor is given by

$$C = \frac{K\varepsilon_0 A}{d}$$

- When a dielectric slab of thickness t and dielectric constant K is introduced between the plate capacitor is given by

$$C = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$$

- When a metallic conductor of thickness t is introduced between the plates, then capacitor is given by

$$C = \frac{\varepsilon_0 A}{d - t}$$

- Capacitors in series :

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_n}$$

- Capacitors in series :

$$\frac{1}{Cp} = \frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_n}$$

- Energy stored in a capacitor :

$$u = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$$

- Energy density : $u = \frac{1}{2}\varepsilon_0 E^2$
- When two capacitors vharged to different potentials are connected by a condicting wire, the common potential,

$$V = \frac{total}{total}\frac{charge}{capacity} = \frac{q_1 + q_2}{C_1 + C_2} = \frac{C_1V_2 + C_2V_2}{2(C_1 + C_2)}$$

  - Energy lost in the process,

$$u_1 - u_2 = \frac{C_1C_2(V_1 - V_2)^2}{2(C_1 + C_2)}$$

CURRENT ELECTRICITY

- Current, $I = \frac{q}{t}$
- Current densitu, $J = \frac{I}{A}$
- Drift velocity of electrons is given by

$$\vec{\upsilon}_d = -\frac{e\vec{E}}{m}\tau\, 2$$

  - Negative sign shows that deift velocity of electrons is in a direction opposite to that of the extemal electroc feld.

- Relationship between current and velocity

$$I = nAe\upsilon_d$$

- Relationship between current density and drift velocity

$$I = ne\upsilon_4$$

- Ohm's law : $V = RI$
- Mobility, $u = \frac{|\upsilon_d|}{E} = \frac{qE\tau / m}{E} = \frac{q\tau}{m}$
- Resistance, $R = \frac{V}{I}$
- Conductance, $G = \frac{1}{R}$
- The resistance of a conductior is

$$R = \frac{m}{ne^2\tau}\frac{1}{A} = \rho\frac{1}{A} \text{ where } = \frac{m}{ne^2\tau}$$

- Condictibvity

$$\sigma = \frac{1}{\rho} = \frac{ne^2\tau}{m} = ne\mu\left[as\mu = \frac{\upsilon_4}{E} = \frac{e\tau}{m}\right]$$

- If the conductor is in the form of wire of length / and a radius r, then its resistance is

$$R = \frac{\rho i}{\pi r^2}$$

- If the conductor has mass m, volume V and density d, then its resistance R is

$$R = \frac{\rho l}{A} = \frac{\rho l^2}{Al} = \frac{\rho l^2}{V} = \frac{\rho l^2 d}{m}$$

- A the cylindrical tube of length / has inner and outer rafii $r_1$and $r_2$respectively. the resistance between its end faces is

$$R = \frac{\rho l}{\pi\left(r_2^2 - r_2^1\right)}$$

- Relationship between J, σ and E

$$J = \sigma E$$

- The resistance of a condictor at temperature t^0C is given by

$$R_1 = R_0(1 + \alpha t + \beta t^{2)}$$

- If R_{t2} and R_{t1} are resistances of the same conductor at temperatures t_1^0C and t_2^0C then

$$R_{t2} = R_{t1}\left[1 + \alpha\left(t_2 - t_1\right)\right]$$

- Resistors in series:

$$R_s = R_1 + R_2 + R_3$$

- Resistors in parallel:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

- Relationship between ε ,V and r

$$r = R\left(\frac{\varepsilon}{V} - 1\right)$$

where ε emf of a cell, r internal resistance and is external resistance.

- Grouping of n cells in series

$$\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2 + ...\varepsilon_n$$

$$r_{eq} = r_1 + r_2 + ...r_n$$

- Grouping of n cells in parallel

$$\varepsilon_{eq} = \varepsilon, r_{eq} + \frac{r}{n}$$

- Wheatstone's bridge,

$$\frac{P}{Q} = \frac{R}{S}$$

- Metre bridge or slide metre bridge,

The unknown resistance,

$$R = \frac{SI}{100 - 1}$$

- Cmparison of emfs of two cells by using popotentiometer,

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{I_1}{I_2}$$

- Determination of internal resistance of a cell by potentiometer,

$$r = \left(\frac{l_1 - l_2}{l_2}\right) R$$

- Electric power, $P = \dfrac{\text{electric workd done}}{\text{time taken}}$

- Electric energy $= \text{Pt} = \text{Vlt} = \dfrac{V^2 t}{R}$

$$\text{P} = \text{VI} = I^2 R = \frac{V^2}{R}$$

MAGNETI CEFFCT OF CURRENT AND MAGNETISM

- Biot Savart's law

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{ldl \sin\theta}{r^2} \text{ or } d\vec{B} = \frac{\mu_0}{4\pi} \frac{(d\vec{l} \times \vec{r})}{r^3}$$

- The magnetic feld B at a point due to a straight wire of finite length carrying current l at a perpedicular distance r is

$$B = \frac{\mu_0 I}{4\pi r}[\sin\alpha + \sin\beta]$$

- The magnetic field at cetre of a circulr coil of radius a carrying current I is

$$B = \frac{\mu_0}{4\pi} \frac{2\pi I}{a} \frac{\mu_0 I}{2a}$$

If the circular coil consists of N turns, then

$$B = \frac{\mu_0}{4\pi} \frac{2\pi NI}{a} \frac{\mu_0 NI}{2a}$$

- The magnetic field at a point on the axis of the ciroular current carrying coil is

$$B = \frac{\mu_0}{4\pi} \frac{2\pi NIa^2}{\left(a^2 + x^2\right)^{3/2}}$$

- Magnetic field at the centre die to current carruing circular arc

$$B = \frac{\mu_0 I \phi}{4\pi}$$

- Ampere's circuital law :

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

- Magnetic field due to an infinitely long straight solid cylindrical wire of radius a, carrying current I
  - Magnetic field at a point outside the wire *i.e.* $(r > a)$ is

$$B = \frac{\mu_0 I}{2\pi r}$$

  - Magnetic field at a point outside the wire *i.e.* $(r > a)$ is
  - Magnetic field at a point outside the wire *i.e.* $(r = a)$ is
- Force on a charged particle in a uniform electric field, $\vec{F} = q\vec{E}$
- Force on a charged particle in a uniform eletric field, $\vec{F} = q(\vec{\upsilon} \times \vec{B})$ or $F = q\upsilon B \sin\theta$
- Motion of a charged particle in a uniform magnetic field
  - Radius of circular path is

$$R = \frac{m\upsilon}{Bq} = \sqrt{\frac{2mK}{q^B}}$$

  - Time period of revolution is

$$T = \frac{2\pi R}{\upsilon} = \frac{2\pi m}{q\text{B}}$$

  - The frequency is, $\upsilon = \dfrac{1}{T} = \dfrac{qB}{2\pi m}$
  - The angular frequency is $\omega = 2\pi\upsilon = \dfrac{q^B}{m}$
- Cyclotron frequency, $\upsilon = \dfrac{Bq}{2\pi m}$

- Force on a current carrying conductor in a uniform magnetic field

$$\vec{F} = I(\vec{I} \times \vec{B}) \text{ or } F = IIB\sin\theta$$

- When two parallel condictors separated separated by a distance r carry currents I1 and I2 the magnetic field of one will exert a force on the other. The force per unit length on eithr conducdtor is

$$f = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r}$$

- The force of attraction or repulsion acting on each conductor of length I due to currents in two parallel conductor is

$$F = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r}I$$

- When two charges q1 and q2 respectively moving with velocities υ_1 and υ_2 are at a distance *r* apart, then the force acting between them is

$$F = \frac{\mu_0}{4\pi}\frac{q_1}{}\frac{q_2}{r^2}\frac{\upsilon_1\upsilon_2}{}$$

- Torque on a current carrying coil placed in a uniform magnetic field

  τ = NIAB sin

  θ = MB sin θ

  - If a is the angle between plane of the coil and the magnetic field, then torque on the coil is

    τ = NIAB casα = MB coaα

- Workdone in rotating the coil through an angle θ from the field direction is

$$W = MB\left(1-\cos\theta\right)$$

- Potential energy of a magnetic dipole

$$U = -\vec{M}\cdot\vec{B} = -MB\cos\theta$$

- An electron revolving around the central nucleus in an stom has a magnetic moment it is given by

$$\vec{\mu}_L = \frac{e}{2m}\vec{L}$$

- Magnetic dipole moment

$$\vec{M} = m(2\vec{l})$$

- The magnetic field due to a ber magnet at any point on the axial line (end on position) is

$$B_{axial} = \frac{\mu_0}{4\pi}\frac{2Mr}{(r^2-l^2)^2}$$

  For short magnet $l^2 << r^2$

  The direction of B $_{\text{axial}}$ is along SN.

- The magnetic field due to a bar magnet at any point on the equatorial line (board-side on position) of the ber magnet is

$$B_{equakuvlal} = \frac{\mu_0 M}{4\pi(r^2+l^2)^{3/2}}$$

  For short magnet $l^2 << r^2$

$$B_{equakuvlal} = \frac{\mu_0 M}{4\pi r^3}$$

  The direction of B_{axial} is parallel to *NS.*

- In moving coil galvanmeter the current *I* passing through the galvanometer is directy proportional to its deflection (θ)

  $I \infty \theta or, I = G\theta$

  where G = $\frac{k}{NAB}$ = galvanometer constant

- Current sensitivity : $I_s = \frac{\theta}{I} = \frac{NAB}{k}$

- Voltage sensitivity : $V_s = \frac{\theta}{V} = \frac{\theta}{IR} = \frac{NAB}{kR}$

- Conversion of galvanometer into ammeter

$$s = \left(\frac{I_8}{I-I_8}\right)G$$

- Conversion of galvanometer into voltmeter

$$R = \frac{V}{I_8} - G$$

- In order to increase the range of voltmeter n times the value of resistance to be connected in series with galvaometer is $R = (n - 1)\,G$.
- When a bar magnet of dipole momet $\vec{M}$ is placed in a uniform magnetic field $\vec{B}$
  - the force on it is Zero
  - the torque on it is $\vec{M} \times \vec{B}$
  - The potential energy is $\vec{M} \cdot \vec{B}$ where we choose the Zero of energy at the orientation when $\vec{M}$ is perpendicular to $\vec{B}$
- Gauss's law for magnetism

$$\oint = \sum_{\substack{\text{all area}\\ \text{elements } \Delta s}} \vec{B} \cdot \Delta\vec{S} = 0$$

- Horizontal component of earth magnetic field

$$B_H = \mathrm{B}\cos\delta$$

- Vertical component of earth's magnetic field

$$B_v = \mathrm{B}\sin\delta$$

$$B = \sqrt{BH^2 + B_v^2} \text{ and } \tan\delta = \frac{B_V}{B_H}$$

- The relationship between magnetic (B) and magnetic intensity (H) is

$$\mathrm{B} = \mu H$$

- Intensity of magnetisation

$$I = \frac{\text{Magnetic moment}}{\text{Volume}} = \frac{M}{V}$$

- Magnetic susceptibility

$$\chi_m = \frac{I}{H}$$

- Magnetic permeabitity

$$\mu = \frac{B}{H}$$

- Relative permeabitity

$$\mu_r = \frac{\mu}{\mu_0}$$

- Relationhip between magnetic permeability and susceptibility

$$\mu_r = 1 + \chi_m \text{ with } \mu_r = \frac{\mu}{\mu_0}$$

- Curie's law: $\chi_m = \dfrac{C}{T}$
- Curie Weiss law : $\chi_m = \dfrac{C}{T - T_c}(T > T_c)$

ELECTROMAGNETIC INDUCTION

- Magnetic flix

$$\phi = \vec{B} \cdot \vec{A} = BA\cos\theta$$

- Faraday's law of electromgnetic induction

$$\varepsilon = -\frac{d\phi}{dt}$$

- When a conducting rod of length I, moves with a velocity υ perpendicular to a uniform magnetic field B, the induced emf across its ends is

$$\varepsilon = Bl\upsilon$$

this is known as motional emf.

- When a conducting rod of length *I*, is rotated perpendicular to a uniform magnetic fild B, then induced emf between the ends of the rod is

$$|\varepsilon| = \frac{B\omega l^2}{2} = \frac{B(2\pi\upsilon)l^2}{2}$$

$$|\varepsilon| = B\upsilon(\pi l^2) = B\upsilon A$$

- When a Current *I* flows through a coil and ϕ is the magnetic flux limked with the coil, then

$$\phi \propto I \text{ or } \phi = LI$$

- The self induced emf is

$$\varepsilon = -\frac{d\phi}{dt} = -L\frac{dl}{dt}$$

- self inductance of a circular coil is

$$\mathrm{L} = \frac{\mu_0 N^2 \pi R}{2}$$

- Let I_p be the current flowing through primary coil at any instant. If ϕ_s is the flux linked with secondary coil then

$$\phi_S \propto I_P \text{ or } \phi_S \propto MI_P$$

Where M is the coefficient of mutual inductance. The emf induced in the secondary coil is given by

$$\varepsilon_s = - M\frac{dI_p}{dt}$$

where M is the coefficient of mutual inductance.

- Coefficient of coupling *(K):*

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

- the coefficient of mutual inductance of two long co-axial solenoifs, each og length *I*, area of cross section *A*, wound on air core is

$$M = \frac{\mu_0 N_1 N_2 A}{I}$$

- Energy stored in an inductor

$$u = \frac{1}{2}LI^2$$

- During the growth of current in a LR circuit is

$$I_0(1-e^{Rt/L}) = (1-e^{-t/\tau})$$

where I_0 is the maximum value of current, τ = l/R = time constant of LR circuit.

- During the decay of current in a LR circuit is

$$I = I_0\left(1-e^{Rt/L}\right) = I_0 e^{-t/\tau}$$

- During charging of capacitor through resistor

$$q = 9_0\left(1-e^{t/RC}\right) = q_0(1-e^{-t/\tau}$$

where q_0 is the maximum value of charge. τ =RC is the time constant of CR circuit.

- Dueing charging of capacitor through resistor

$$q = q_0 e^{-t/RC} = q_0 e^{-t\lambda}$$

ALTERNATING CURRENT

- Alternating current can be represented by a sine curve or a cosine curve
I= I_0 sin ωt or I= I_0cis ωt

where $$\omega = \frac{2\pi}{T} = 2\pi$$

- Mean or average value of alternating current or voltage over one complete cycle

$$I_m \text{ or } \overline{I} \text{ or } I_m = \frac{\int_0^t I_0 \sin dt}{\int_0^r dt} = 0$$

$$V_m \text{ or } \overline{V} \text{ or } V_{av} = \frac{\int_0^T V_0 \sin dt}{\int_0^T dt} = 0$$

- Average value of alternating current for second cycle is

$$I_{av} = \frac{\int_0^{T/2} V_0 \sin \omega dt}{\int_0^{T/2} dt} = \frac{2I_0}{\pi} = 0.637 I_0$$

Similarly, for alternating voltage, the average value over second half cycle is

$$Vav = \frac{\int_0^{T/2} V_0 \sin \omega dt}{\int_0^{T/2} dt} = \frac{2V_0}{\pi} = 0.637 V_0$$

- Average value of alternating current for second cycle is

$$I_{av} = \frac{\int_{T/2}^{T} I_0 \sin \omega dt}{\int_{T/2}^{T} dt} = \frac{2I_0}{\pi} = 0.637 I_0$$

Similarly, for alternating voltage, the average value over second half cycle is

$$Va\upsilon = \frac{\int_{T/2}^{T} V_0 \sin \omega dt}{\int_{T/2}^{T} dt} = \frac{2V_0}{\pi} = 0.637V_0$$

❑ Mean value or a average value of altrenating current over any half cycle

$$Ia\upsilon = \frac{2I_0}{\pi} = 0.637I_0$$

similarly, for alternating voltage

$$Va\upsilon = \frac{2V_0}{\pi} = 0.637V_0$$

❑ Root mean square rms value of alternating current

$$I_{rms} \text{ or } I_\upsilon = \frac{1_0}{\sqrt{2}} = 0.707I_0$$

similarly, for alternating voltage

$$V_{rms} = \frac{V_0}{\sqrt{2}} = 0.707V_0$$

❑ Form factor $= \frac{I_{rms}}{I_{a\upsilon}}$

❑ Inductive reactance,

$$\chi_L = \omega L = 2\pi\upsilon L$$

❑ Capacitive reactance, xc = $\frac{1\omega}{\omega C} = \frac{1}{2\pi\upsilon C}$

❑ The impedance of the series LCR circuit

$$z = \sqrt{R^2 + (X_1 - X_c)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

❑ Admittance = $\frac{1}{impedance}$ or Y= $\frac{1}{Z}$

❑ Susceptance = $\frac{1}{\text{reactance}}$

❑ Inductiive susceptance = $\frac{1}{\text{inductive reactance}}$

or $S_L = \frac{1}{X_L} = \frac{1}{\omega_L}$

❑ Capacitive susceptance

$$= \frac{1}{\text{capacitive reactance}}$$

or $S_c = \frac{1}{X_c} = \frac{1}{1/\omega C} = \omega C$

❑ The resonant frequency is

$$\upsilon_r = \frac{1}{2\pi\sqrt{LC}}$$

$$\omega_r = \frac{1}{\sqrt{LC}}$$

❑ Quality factor,

$$Q = \frac{X_L}{R} = \frac{\omega_r L}{R}$$

$$Q = \frac{X_c}{R} = \frac{1}{\omega_r CR}$$

$$Q = \frac{1}{R} = \sqrt{\frac{L}{C}}$$

❑ Average power,

$$P_{a\upsilon} = P_{a\upsilon} V_{rms} I_{rms} = \frac{V_0 I_0}{2} \cos\phi$$

❑ Apparent power, $V_\upsilon V_{rms} I_{rms} = \frac{V_0 I_0}{2}$

❑ Efficiency of a transformer,

$$\eta = \frac{\text{output power}}{\text{input power}} = \frac{V_s I_s}{V_p I_p} \frac{power}{power}$$

❑ The efficiency of a dc motor is given by

$$\eta = \frac{\text{back emf}}{\text{emf of battery}}$$

ELECTROMAGNENTIC WAVES

- The displacement current is give by

$$I_D = \varepsilon 0 \frac{d\phi_E}{dt}$$

- Four Maxwell's equations are :
  - Gauss's iaw for electrostatics

$$\oint \bar{E}.d\bar{S} = \frac{q}{\varepsilon_0}$$

  - Gauss's law for magnetism

$$\oint \bar{B}.d\bar{S} = 0$$

  - Faraday's law of electromagnetic induction

$$\oint \vec{E}.d\vec{I} = \frac{d\phi_B}{dt}$$

  - Maxwell-Ampere's circuital law

$$\oint \vec{B}.d\vec{I} = \mu_0 \left[1 + \varepsilon_0 \frac{d\phi_E}{dt}\right]$$

- The amplitudes of electeic and magnetic fields in free space, in electromagnetic waves are related by

$$E_0 = cB_0 \text{ or } B_0 = \frac{E_0}{c}$$

- The speed of electromagnetic wave in free space is

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

- The speed of electromagnetic wave in free space is

$$\upsilon = \frac{1}{\sqrt{\mu\varepsilon}}$$

- The energy density of the electric field is

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

- The energy density of magnetic fields

$$u_B = \frac{1}{2}\frac{B^2}{\mu_0}$$

- Average energy density of the electric field is

$$< u_E > = \frac{1}{4}\varepsilon E_0^2$$

- Average energy density of the electric field is

$$< u_B > = \frac{1}{4}\frac{B_0^2}{\mu_0} = \frac{1}{4}\varepsilon_0 E_0^2$$

- Average energy density of the electric field is

$$< u > = \frac{1}{2}\varepsilon_0 E_0^2$$

- Intensuty of electromagnetic wave

$$I = < u > c = \frac{1}{2}\varepsilon_0 E_0^2 c$$

- Momentum of electromagnetic wave

$$p = \frac{u}{c} \text{ (complete absorption)}$$

$$p = \frac{2u}{c} \text{ complete reflection}$$

- The poynting vector is

$$\vec{S} = \frac{1}{\mu_0}\left(\vec{E} \times \vec{B}\right)$$

RAY OPTICS

- When two plane mirrors are inclined at an angle θ and an object is placed between them, the number of images of an object are formed due to multiple reflections.

$n = \frac{360^0}{\theta}$	Position of object	Nimber of images
even	anywhere	$n - 1$
odd	symmetric	$n - 1$
	asymmetric	n

- If $\frac{360^0}{\theta}$ is a fraction, the number of images formed will be equal to its integral parrt.

- If $\frac{360^o}{\theta}$ is a fraction, the number of images formed will be equal to its integral part.
- Sign convntions :
  - All distances have to be measured from the pole of the mirror.
  - Distances measured in the the direction of incident light are positive, and those mea sured in opposite direction aretaken as negetive.
  - Heights measured upwards and normal to the principal axis of the mirror are taken as positive, while those measured down wards are taken as negetive.
- The focal length of a spherical mirror of radius *R* is given by

$$f = \frac{R}{2}$$

- Transverse or linear magnification

$$m = \frac{\text{size of image}}{\text{size of object}} = \frac{v}{u}$$

- Longitudinal magnification

$$m_L = -\frac{d\upsilon}{du}$$

- Superficial magnification

$$m_s = \frac{\text{area of image}}{\text{area of object}} = m^2$$

- Mirrors formula : $\frac{1}{u} + \frac{1}{\upsilon} = \frac{1}{f}$
- Newton's formula : $f_2 = \chi$ y
- Laws of refraction : $\frac{\text{sin i}}{\text{sin r}} = 1\mu_2$
- Absolute refractive index

$$1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{\left(\frac{c}{\upsilon_2}\right)}{\left(\frac{c}{\upsilon_1}\right)} = \frac{\upsilon_1}{\upsilon_2}$$

- Lateral shift, $d = \frac{\text{t sin } (i - r)}{\text{cos r}}$
- If there is an ink spot at the bottom of a glass slab, it appears to be raisedby a distance

$$d = t - \frac{r}{\mu} = t\left(1 \frac{1}{\mu}\right)$$

- Critical angle : It is that angle of incidence for which the angle of refraction becomes 90_0 It is given by

$$\sin i_C = \frac{1}{R_{\mu D}}$$

It the medium is air or vacuum, then

$$\sin i_c = \frac{1}{\mu}$$

- A diver in weter at a depth d sees the world outside through a horizontal circle of radius

$$r = s \tan i_c = \frac{d}{\sqrt{\mu^2 - 1}}$$

- When the object is situated in rarer medium, the relation between μ_1(refractive inbox of refracting surface) and R (radius of curvature) with the objecr and image distacrs is given by

$$-\frac{\mu_1}{\mu} + \frac{\mu_2}{\mu} = \frac{\mu_2 - \mu_1}{R}$$

- When the object is situated in denser medium, the relation between μ_1, μ_2, R, u and υ cwn be obtained by interchanging $\mu_1$and μ_2. In that case, the relation becomes

$$-\frac{\mu_2}{u} + \frac{\mu_1}{\upsilon} = \frac{\mu_1 - \mu_2}{R} \text{ or } -\frac{\mu_2}{\upsilon} + \frac{\mu_2}{u} = \frac{\mu_2 - \mu_2}{R}$$

- Lens maler's formula

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

- Thin lens formula $\frac{1}{\upsilon} - \frac{1}{u} = \frac{1}{f}$
- Linear magnification

$$m = \frac{\text{size of image (I)}}{\text{size of object(O)}} = \frac{\upsilon}{u}$$

- Power of a lens

$$P = \frac{1}{\text{focal length in metres}}$$

- Combination of thin lenses in contact

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + ...$$

- The total power of the combination is given by

$$P = P_1 + P_2 + P_3 + ...$$

- The total megnification of the combination is given by $m = m_1 \times m_2 \times m_3 .....$
- When two thin lenses of focal lengths fr and placed coaxially and separated by a distance d, the focal length of a combination is give by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

- In terms of power $P_1 + P_2 - dP_1P_2$
- The refractive index of the material of the prism is

$$\mu = \frac{\sin\left[\frac{(A+\delta_m)}{2}\right]}{\sin\left(\frac{A}{2}\right)}$$

where A is the angle of prism and δ_m is the angle of minimum deviation.

- Mean deviation, $\delta = \frac{\delta_y + \delta_R}{2}$
- Diapersive power,

$$\omega = \frac{\text{angular dispersion}\left(\delta_y - \delta_R\right)}{\text{mean deviation }(\delta)}$$

$$\omega = \frac{\mu_{y-}\mu_R}{(\mu - 1)}$$

where $\mu = \frac{\mu_{y-}\mu_R}{2}$ mean refractive index

- Simple microscope
  - Magnifying power of simple microscope

$$M = \frac{\text{angle cubtended by inage at the eue}}{\text{angle subended by the object at the eye}}$$

$$= \frac{\tan\beta}{\tan\alpha} = \frac{\beta}{\alpha}$$

  - When the image is formed at infinity (fae point),

$$M = \frac{D}{f}$$

  - When the image is formed at the least distance of distinct vision D (near point),

$$M = 1 + \frac{D}{f}$$

- Compound microscope
  - Magnifying power of a compound micrscope

$$M = m_0 \times m_e$$

  - When the final image is formed at infinity (normal adjustment),

$$M = \frac{\upsilon_0}{u_0}\left(\frac{D}{f_e}\right)$$

Length of tube, $L = \upsilon_0 + f_e$

  - When the final image is formed at least distance of distinct vision,

$$M = \frac{\upsilon_0}{u_0}\left(1 + \frac{D}{f_e}\right)$$

Where $u_0$and υ_0 represent the distance of object and image from the objective lens, f_e is focal length of an eye lens.

Length of the tube, $L = \upsilon_0 + \left(\frac{f_e D}{f_e + D}\right)$

- Astronomical telescope (Refracting type) When the final image is formed at infinity

- Magnifying power, $M = \frac{f_0}{f_e}$
- Length of tube is $L = f_0 + f_e$
  When the image is formed at least distance of distinct vision
- Magnifying power, $M = \frac{f_0}{f_e}\left(1+\frac{f_e}{D}\right)$
- Length of tube is $L = f_0 + \left(\frac{f_e D}{f_e + D}\right)$

- Reflecting type telescope
  - Magnifying power, $M = \frac{f_0}{f_e} = \frac{(R/2)}{f_e}$

WAVE OPTICS

- If a, b are the amplitudes of interfering waves due to two coherent sources and ϕ is constant phase difference between the two waves at any point P, then the resultant amplitude at P will be

$$R = \sqrt{a^2 + b^2 + 2\text{ ab}\cos\phi}$$

- For constructive interference (i.e.formation of bright fringes)
  - For n[th] bright fringe,

    path difference = $x_n \frac{d}{D} = n\lambda$

    where n = 0 for central bright fringe
    n = 1 for first bright fringe,
    n = 2 for second bright fringe and so on
    d = distance between the two slits
    D = distance of slits from the screen
    x_n = distance of n[th] bright fringe from the centre.

    $$\therefore x_n = n\lambda\frac{D}{d}$$

- For destructive interference (i.e. formation of dark fringes).
  - For n[th] bright fringe,

    path difference = $x_n = \frac{d}{D} = (2n-1)\frac{\lambda}{2}$

    where n = 1for first dark fringe,
    n = 2 for 2[nd] dark fringe and so on.
    x_n = distance of nth dark fringe from the centre

    $$\therefore x_n = (2n-1)\frac{\lambda D}{2d}$$

- Fringe width, $\beta = \frac{\lambda D}{d}$
- Angular fringe width, $\theta = \frac{\beta}{D} = \frac{\lambda}{d}$
- If Wv W2are widths of slits, Iv I2 are intensities of lifht coming from two slits; a, b are amplitudes of light from these slits, then

$$\frac{W_1}{W_2} = \frac{I_1}{I_2} = \frac{a^2}{b^2}$$

$$\frac{I_{max}}{I_{min}} = \frac{(a+b)^2}{(a-b)^2}$$

- Fringe vidibility, $V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$
- When entire apparatus of Young's double slit experiment is innersed in a medium of refeactive index μ ,then fringe width becomes

$$\beta' = \frac{\lambda D}{d} = \frac{\lambda D}{\mu d} = \frac{\beta}{\mu}$$

- When a thin transparent plate of thickness t and refractive index μ is placed in the path of one of the interfeing waves, fringe width remains unaffected but the entire pattern shifts by

$$\Delta x = (\mu - 1)t\frac{D}{d} = (\mu - 1)t\frac{\beta}{\lambda}$$

- Diffraction due to a single slit
  - Condition for n^{th} secondary maximum is

    path difference=$a\sin\theta_n$=(2n+1)$\frac{\lambda}{2}$

where n = 1 ,2, 3, ...

- Condition for n^{th} secondary maximum is

  Path difference = $a \sin\theta_n = n\lambda$

  where n = 1, 2, 3,.......

  width of secondary maxima or minima

$$\beta = \frac{\lambda D}{a} = \frac{\lambda f}{a}$$

where

a = width of slit

D= distance of scren from the slit

f = focal length of lens for diffracted light

- width of central maximum= $\frac{2\lambda D}{a} = \frac{2f\lambda}{a}$
- Angular fring width of cental maximum= $\frac{2\lambda}{a}$
- Angular fring width of secondary maxima or minima = $\frac{\lambda}{a}$
- Fresnel distance,

$$Z_F = \frac{a^2}{\lambda}$$

- Resolving power of a microscope

$$= \frac{1}{d} = \frac{2\mu\sin\theta}{\lambda}$$

- Resolving power of a

$$= \frac{1}{d\theta} = \frac{D}{1.22\lambda}$$

- Laws of malus :

$$I = I_0 \cos^2\theta$$

- Brewster's law : $\mu = \tan i_p$

DUAL NATURE OF RADIATION AND MATTER

- Energy of a photon,

$$E = h\upsilon = \frac{hc}{\lambda}$$

- Momentum of photon is

$$p = \frac{E}{c} = \frac{h\upsilon}{c}$$

- The moving mass of photon is

$$m = \frac{E}{c^2} = \frac{h\upsilon}{c^2}$$

- Stopping potential
- $K_{max} = eV_0 = \frac{1}{2} m\upsilon_{max}^2$
- Einstein's photoelectric equation :

  if a light of frequency υ is incident on a photo-sensitive maximum having work function (ϕ_0), then maximum kinetic cnergy of the emitted electron is given as $K_{max} = h\upsilon - \phi_0$

  for $\upsilon > \upsilon_0$ or $eV = h\upsilon - \phi_0 = h\upsilon - h\upsilon_0$

$$eV_0 = K_{max} = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

- The de Broglie wavelength associated with a moving particle,

$$\lambda = \frac{h}{p} = \frac{h}{m\upsilon}$$

  - If the rest mass of a particle is m_0 its de Broglie wavelength is given by

$$\lambda = \frac{h\left(1 - \frac{\upsilon^2}{c^2}\right)^{1/2}}{m_0\upsilon}$$

- In terms of kinetic energy K, de Broglie wave-length is given by $\lambda = \frac{h}{\sqrt{2mK}}$
- If a particleof charge q is accelerated through a potential difference V, its de Broglie wave-length is given by $\lambda = \frac{h}{\sqrt{2mV}}$

  For an electron, $\lambda = \left(\frac{150}{V}\right)^{1/2} \overset{0}{A}$

❍ For a gas molecule of mass m at temperature T lelvin, its de Broglie wavelength is given by $\lambda \frac{h}{\sqrt{3\text{mkT},}}$ where k is the Boltzmann constant.

ATOMS AND NUCLEI

❑ Rutherford scattering formula

$$\text{N}(\theta) = \frac{\text{N}_i ntZ^2e^4}{(8\pi_0)^2 r^2K^2 \sin^4(\theta/2)}$$

❑ The fraction of incident alpha particles scattered by an angle θ or greater

$$\text{f} = \pi nt\left(\frac{Ze^2}{4\pi\varepsilon_0 k}\right)^2 \cot^2\frac{\theta}{2}$$

❑ The scattering angle θ of the α particle and impact parameter b are related as

$$\text{b} = \frac{Ze^2\cos(\theta/2)}{4\pi\varepsilon_0 K}$$

❑ Distance of closest approach

$$r_0 = \frac{2Ze^2}{4\pi\varepsilon_0 K}$$

❑ Angular momentum of the electron in a stationary orbit is an integral multiple of h/2 π .

i.e., $$\text{L} = \frac{nh}{2\pi} \text{ or } m\upsilon r = \frac{nh}{2\pi}$$

❑ The frequency of a radiation

$$\upsilon \frac{E_2 - E_1}{h}$$

❑ Bohr's formulae

❍ Radius of n^{th} orbit

$$r_n = \frac{4\pi\varepsilon_0 n^2h^2}{4\pi^2 mZe^2}\ ;\ r_n = \frac{0.53n^2}{Z}\overset{0}{\text{A}}$$

❑ Velocity of electron in the n^{th} orbit

$$\upsilon_n = \frac{1}{4\pi\varepsilon_0}\frac{Z_{e^2}}{nh} = \frac{2.2\times10^6 Z}{n} ms^{-1}$$

❍ The kinetic energy of the electron in the n^{th} orbit

$$\text{Kn} = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{2r_n} = \left(\frac{1}{4\pi\varepsilon_0}\right)^2\frac{2\pi^2me^4Z^2}{n^2h^2}$$

$$= \frac{13.6Z^2}{n^2}eV$$

❍ The potential energy of electron in n^{th} orbit

$$u_n = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r_n} = -\left(\frac{1}{4\pi\varepsilon_0}\right)^2\frac{4\pi^2me^4Z^2}{n^2h^2}$$

$$= -\frac{27.2Z^2}{n^2}e.V$$

❍ Total energy of electron in n^{th} orbit

$$E_n = u_n + k_n = -\left(\frac{1}{4\pi\varepsilon_0}\right)^2\frac{2\pi^2me^4Z^2}{n^2h^2}$$

$$= -\frac{13.6Z^2}{n^2}eV$$

❍ Frequency of electron in n_{th} orbit

$$\upsilon_n = \left(\frac{1}{4\pi\varepsilon_0}\right)^2\frac{4\pi^2Ze^4m}{n^3h^3} = \frac{6.62\times10^{15}Z^2}{n^3} \text{ z}$$

❍ Wavelength of radiation in the transition from

$n_2 \rightarrow n_1$ is give by

$$1\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n_2^2}\right]$$

Where R is called Rydberg's constant.

$$R = \left(\frac{1}{4\pi\varepsilon_0}\right)^2\frac{2\pi^2me^4}{ch^3} = 1.097\times10^7 m^{-1}$$

❑ **Lyman series :**

❍ Emission spectral lion from jigher energy levels (n_2 = 3, 3,.... ∞) to first energy level (n_1 = 1) consitipn of electute Lyman series.

$$\frac{1}{\lambda}=R\left[\frac{1}{1^2}-\frac{1}{n_2^2}\right]$$

- **Balmer series:**
  - Emission spectral lion from jigher energy levels (n_2 = 3, 4,.... ∞) to first energy level (n_1 = 2)
    consitipn of electute Lyman series.

$$\frac{1}{\lambda}=R\left[\frac{1}{2^2}-\frac{1}{n_2^2}\right]$$

- **Paschen series:**
  - Emission spectral lion from jigher energy levels (n_2 = 4, 5,.... ∞) to first energy level (n_1 = 3)
    consitipn of electute Lyman series.

$$\frac{1}{\lambda}=R\left[\frac{1}{3^2}-\frac{1}{n_2^2}\right]$$

- **Brackett series:**
  - Emission spectral lion from jigher energy levels (n_2 = 5, 6,7,.... ∞) to first energy level (n_1 = 4)
    consitipn Brackett series.

$$\frac{1}{\lambda}=R\left[\frac{1}{4^2}-\frac{1}{n_2^2}\right]$$

- **Pfund series:**
  - Emission spectral lines corresponding to the transition of electron fron higher energy (n1=4) constitute Braclett series.

$$\frac{1}{\lambda}=R\left[\frac{1}{5^2}-\frac{1}{n_2^2}\right]$$

- Number of spectral lines due to transition of electron from n^{th} orbit to lower orbit is electron n^{th} orbit to lower orbit is

$$N=\frac{n(n-1)}{2}$$

- Ionization potential = $\frac{13.6Z^2}{n^2}V$
- Ionization potential = $\frac{13.6Z^2}{n^2}V$
- Energy quantisation,

$$E_n=\frac{n^2h^2}{8mL^2} \text{ where n = 1,2,3,.........}$$

- **Nuclear radius,** $R=R_0A^{1/3}$
  Where R_0 is a constant and A is the mass number.
- Nuclear density,

$$P=\frac{\text{nuclear mass}}{\text{volume of mucleus}}$$

- Mass defect is give by

$$\Delta m=\left[Zm_p+(A-Z)m_n-m_N\right]$$

- The binding energy of nucleus is given by

$$E_b=\Delta mc^2=\left[Zm_p+(A-Z)m_n-m_n\right]c^2$$

$$\left[Zm_p+(A-Z)m_N-m_N\right]\times 931.49MeV/u.$$

- The binding energy per nucleon of a nucleus

$$=\frac{E_b}{A}$$

- Packing fraction

$$=\frac{\text{mass cxcess}}{\text{mass number}}=\frac{M-A}{A}$$

- Low of radioactive decay

$$=\frac{DN}{dt}=\lambda N(t)N(t)=N_0e^{-\lambda t}$$

- Half-life of a radioactive substance is given by

$$T_{1/2}=\frac{In2}{\lambda}=\frac{0.693}{\lambda}$$

- Mean life or average lift of a radioactive substance is given by

$$\tau=\frac{T_{1/2}}{0.693}=1.44T_{1/2}$$

- Activity:

R= $\frac{dN}{dt}$

- Activity law : $R(t) = R_0 e^{\lambda\tau}$

  where $R_0 = \lambda N_0$ is the decay rate at t = 0 and $R = N\lambda$.
- Fraction of nuclei left undecayed after *n* half lives is

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{t/T_{1/2}} \text{ or } t = nT_{1/2}$$

- Alpha decay: It is represented by

$$\overset{A}{Z} X \rightarrow \overset{A-4}{Z-2} Y + \overset{4}{2} He$$

- Beta decay: It is represented by

$$\overset{A}{Z} X \rightarrow \overset{A}{Z+1} Y + \bar{e} + \bar{\upsilon}$$

- Gamma decay: It is represented by

$$\overset{A}{Z} X \rightarrow \overset{A}{Z} + \lambda$$

  denotes the excited nuclear state.
- **Nuclear reaction :**
  - It is represented by

    $A + a \rightarrow B + b + Q$
  - Q value of nuclear reaction

    Q = $(m_A + m_a - m_B - m_b)c^2$
- Neutron represented factor (K)

$$= \frac{\text{rate of production of neutrons}}{\text{rate of loss of neutrons}}$$

SEMICONDUCTOR DEVIVES

- Forbidden energy gap

$$E_g = h\upsilon = \frac{hc}{\lambda}$$

- The intrinsic concentration ni varies with temperature T as

$$n_i^2 = A_0 T^3 e^{-EgKT}$$

- The conductivity of the semiconductor is given by

$$\sigma = \frac{1}{2} = e(n_e \mu_h)$$

where μ_e and n_h are the electron and hole mobilities, μ_e and n_h are the electron and hole densities, e is the electronic charge.

- The conductivity of an intrinsic semiconductor is

$$\sigma_i = n_{ie}(\mu_e + \mu_h)$$

- The conductivity of n-type semiconductor is

$$\sigma_i = eN_d \mu_e$$

- The conductivity of n-type semiconductor is

$$\sigma_p = eN_a \mu_h$$

- The current in the junction diode is given by

$$I = I_a(e^{eV/KT} - 1)$$

where k = Boltzmann constant, I0 = reverse saturation current.

In forward biasing, V is positive and low, $e^{eVKT} >> 1$, then forward current,

$$I_f = I_0\left(e^{eVKT}\right)$$

In reverse biasing, V is negative and high $e^{eVKT} <<< 1$, low,then reverse current,

$$I_f = I_0$$

- Dynamic resistance,

$$r_d = \frac{\Delta V}{\Delta I}$$

- Half wave rectifier:
  - Peak value of current is

$$I_m = \frac{Vm}{rf + R_L}$$

where R_f is the forward diode resistance, R_L is the load resistance and V_m is the peak value of the alternating voltage.

  - rms value of current is

$$L_{rms} = \frac{I_m}{2}$$

- dc value of current is

$$I_{dc} = \frac{I_m}{\pi}$$

- Peak inverse voltage is

$$P.I.V = V_m$$

- dc value of voltage is

$$V_{dc} = I_{dc} R_L = \frac{I_m}{\pi} R_L$$

- Full wave rectifier,
  - Peak inverse voltage is

$$I_m = \frac{V_m}{r_f + R_L}$$

  - dc value of current is

$$I_{dc} = \frac{21_m}{\pi}$$

  - rms value of current is

$$I_{rms} = \frac{1_m}{\sqrt{2}}$$

  - Peak inverse voltage is

$$P.I.V = 2V_m$$

  - dc value of voltage is

$$V_{dc} = I_{dc} = \frac{2I_m}{\pi} R_L$$

- Ripple frequency,

$$r = \frac{\text{rms value of the components of wave}}{\text{average or dc value}}$$

$$r = \sqrt{\left(\frac{I_{rms}}{I_{dc}}\right)^2 - 1}$$

- For half wave rectifier,

$$I_{rms} = \frac{I_m}{2}, I_{dc} = \frac{I_m}{\pi}$$

$$r = \sqrt{\left(\frac{I_m / 2}{I_m / \pi}\right)^2 - 1} = 1.21$$

- For Full wave rectifier,

$$I_{rms} = \frac{I_m}{\sqrt{2}}, I_{dc} = \frac{2I_m}{\pi}$$

$$r = \sqrt{\left(\frac{I_m / \sqrt{2}}{2I_m / \pi}\right)^2 - 1}$$

$$= 0.482$$

- Rectification efficiency,

$$\eta = \frac{\text{dc power delivered to load}}{\text{ac input power transformer secondary}}$$

- For half wave rectifier,
  - dc power delivered to the load is

$$P_{dc} = I_{dc}^2 R_L = \left(\frac{I_m}{\pi}\right)^2 R_L$$

  - Inout ac power is

$$P_{dc} = I_{rms}^2 \left(r_f + R_L\right) \left(\frac{I_m}{\pi}\right)^2 \left(r_f + R_L\right)$$

- Rectification efficiency,

$$\eta = \frac{P_{dc}}{P_{ac}} = \frac{\left(I_m / \pi\right)^2 R_L}{\left(I_m / 2\right)^2 \left(r_f + R_L\right)} \times 100\%$$

- For a full wave rectifier,
  - dc power delivered to the load is

$$P_{dc} = I_{dc}^2 R_L = \left(\frac{2I_m}{\pi}\right)^2 R_L$$

  - Inout ac power is

$$P_{dc} = I_{rms}^2 \left(r_f + R_L\right) \left(\frac{I_m}{\sqrt{2}}\right)^2 \left(r_f + R_L\right)$$

- Rectification efficiency,

$$\frac{P_{dc}}{P_{ac}} = \frac{\left(2I_m / \pi\right)^2 R_L}{\left(I_m / \sqrt{2}\right)^2 \left(r_f + R_L\right)} \times 100\%$$

$$= \frac{81.2}{1 + r_f / R_L} \%$$

- Form factor $= \frac{I_{rms}}{I_{dc}}$

- For half wave rectifier,

$$I_{rms} = \frac{I_m}{2}, I_{dc} = \frac{I_m}{\pi}$$

- Form factor $= = \frac{I_m / 2}{I_m / \pi} = \frac{\pi}{2} = 1.57$
- For Full wave rectifier,

$$I_{rms} = \frac{I_m}{\sqrt{2}}, I_{dc} = \frac{2I_m}{\pi}$$

Form factor $= \; I_{rms} = \frac{I_m / \sqrt{2}}{2I_m / \pi} = \frac{\pi}{2\sqrt{2}} = 1.11$

- Common emitter amplifier:
  - dc current gain,

$$\beta_{dc} = \frac{I_c}{I_{\mathrm{B}}}$$

  - ac current gain,

$$\beta_{ac} = \frac{\Delta I_c}{\Delta I_{\mathrm{B}}}$$

  - Voltage gain,

$$A_\upsilon = \frac{V_0}{V_i} = -\beta_{ac} \times \frac{R_0}{R_i}$$

  - Power gain,

$$Ap = \frac{\text{output power}(\mathrm{p}_0)}{\text{input power }(\mathrm{P}_i)} = \beta_{ac} \times A_\upsilon$$

  - Voltage gain,(in dB) = 20 $\log_{10} \frac{V_0}{V_i}$

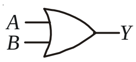
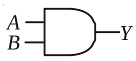
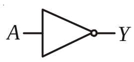
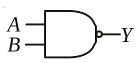
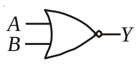
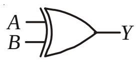
    = 20 $\log_{10} A\upsilon$
  - Power gain (in dB) =10 log $\frac{P_0}{P_i}$
  - Common base amplifier:
  - dc current gain, $\alpha_{dc} = \frac{I_c}{I_E}$
  - ac current gain, $\alpha_{ac} = \left(\frac{\Delta I_c}{\Delta I_E}\right)$
  - Voltage gain, $A_p = \frac{V_0}{V_1} = \alpha_{ac} \times \frac{R_0}{R_1}$
  - Power gain

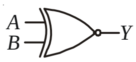
$$A_p = \frac{\text{output power}(\mathrm{P}_0)}{\text{input power }(\mathrm{P}_i)} = \alpha_{ac} \times A_\upsilon$$

  - Relationship between α and β

$$\beta = \frac{\alpha}{1-\alpha}; \alpha = \frac{\beta}{1+\beta}$$

The frequency of the oscillation of a *LC* oscillator is $\upsilon = \frac{1}{2\pi\sqrt{LC}}$

Name of gate	**Symbol**	**Truth Table**	**Boolean expression**
OR		A B Y 0 0 0 0 1 1 1 0 1 1 1 1	Y = A + B
AND		A B Y 0 0 0 0 1 0 1 0 0 1 1 1	Y = A.B
NOT		A Y 0 1 1 0	$Y = \overline{A}$
NAND		A B Y 0 0 1 0 1 1 1 0 1 1 1 0	$Y = \overline{A \cdot B}$
NOR		A B Y 0 0 1 0 1 0 1 0 0 1 1 0	$Y = \overline{A + B}$
XOR		A B Y 0 0 0	$Y = A \cdot \overline{B} + \overline{A} \cdot B$

XNOR		0 1 1 1 0 1 1 1 0 A B Y 0 0 1 0 1 0 1 0 0 1 1 1	$Y = A \cdot B + \overline{A} \cdot \overline{B}$

❑ De morgan's theorems

- $\overline{A+B} = \overline{A} \cdot \overline{B}$
- $\overline{A \cdot B} = \overline{A} + \overline{B}$

❑ Boolean identities :

$A + B = B + A$	$A \,.\, B = B.A$
$A + (B + C) = (A+B)+C$	$A \,.\, (B+C) = (A.B).C$
$A + (B + C) = A.B+A.C$	$A + B.C=(A+B)(A+C)$
$A + 0 + A$	$A \,.1= 0$
$A + 1 + 1$	$A \,.\, 0 = 0$
$A + A + A$	$A \,.\, A = A$
$A + \overline{A} = 1$	$A \cdot \overline{A} = 1$
$\overline{\overline{A}} = A$	$\overline{\overline{A}} = A$
$\overline{A+A} = \overline{A} \cdot \overline{B}$	$\overline{A \cdot A} = \overline{A} + \overline{B}$
$A + A \cdot B = A$	$A \cdot (A+B) = A$
$A + \overline{A} \cdot B = A + B$	$A \cdot (\overline{A} + B) = A \cdot B$

COMMUNICATION SYSTEM

❑ Critical frequency, $\upsilon_c = 9(N_{\max})^{1/2}$

where $N_{\max}$ the maximum number density of electron per m^3.

❑ Maximum usable frequency

$$MUF = \frac{\upsilon_c}{\cos i} = \upsilon_c \sec i$$

where i is the angle between normal and the direction of incidence of waves.

❑ The slip distance is given by

$$D_{kip} = 2h\sqrt{\left(\frac{\upsilon_0}{\upsilon_c}\right)^2 - 1}$$

where h is the height of reflecting later of atmosphere, υ_0 = maximum frequency of elecreomagnetic waves used and υ_c is the critical frequency for that lyer.

❑ If h is the transmitting antenna, then the distance to the horizon is given by

$$d = \sqrt{2hR}$$

where R is the raadius of the earth.
For TV signal,

Area covered = $\pi d^2 = \pi hR$

Population covered = population density $\times$ area covered ce d_m between two antennas having heights h_T and h_R above the earth is given by

$$d_m = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$

Where h_T is the height of the transmitting antenna and h_R is thr height of the receiving antenna and R is the radius of the earth.

❑ The amplitude modulated signal is represented as $c_m(t) = \left(A_c + A_m \sin \omega_m t\right)\sin \omega_m t$

where $\omega_c = 2\pi\upsilon_c$

$\omega_m = 2\pi\upsilon_m$

$\mu = \frac{A_m}{A_c}$ is the modulation index.

❑ The amplitude modulated signal contains three frequencies, viz. $\upsilon_c \upsilon_c \upsilon_c + \upsilon_m$ and $(\upsilon_c - \upsilon_m)$ the first ferquency is the carrier frequency. Thus, the process of modulation does not change the original carrier frequency but produces two new frequencies $\upsilon_c + \upsilon_m$ and $\upsilon_c - \upsilon_m$ which are known as sideand frequencies

$$\upsilon_{SB} = \upsilon_c \pm \upsilon_m$$

- Frequency of lower side band

$$\upsilon_{LSB} = \upsilon c\text{-}\upsilon_m$$

- Frequency of higher side band

$\upsilon_{USB} = \upsilon_c - \upsilon_m$

- Bandwidth of AM signal =

$\upsilon_{USB} = -\upsilon_{USB} == 2\ \upsilon_m$

- Average power per cycle in the carrier wave is

$$P_c = \frac{A_c^2}{2R}$$

where R is the resistance

- Total power per cycle in the modulated wave

$$P_t = P_c\left(1+\frac{\mu^2}{2}\right)$$

- It I, is rms value of total modulated current and I_c is the rms value of unmodulated carrier current, then

$$\frac{I_t}{I_c} = \sqrt{1+\frac{\mu^2}{2}}$$

$\upsilon_{\max} = \upsilon_c + \frac{kV_m}{2\pi} and \upsilon_c = \upsilon_c - \frac{kV_m}{2\pi}$ etection of *AM* wave, the essential condition is

$$\frac{1}{\upsilon_c} =<< RC$$

- The instantaneous frequency of the frequency modulated wave is

$$\upsilon = \upsilon_c + \frac{V_m}{2\pi}\sin\omega_m^t$$

where k is the proportionality condtant.

- The maximum and minimum values of the frequency is

$$\upsilon_{\max} = \upsilon_c + \frac{kV_m}{2\pi} and \upsilon_c = \upsilon_c - \frac{kV_m}{2\pi}$$

- Frequency deviation,

$$\delta = \upsilon_{\max} - \upsilon_c - \upsilon_{\min} = \frac{kV_m}{2_\pi}$$

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UNITS AND MEASUREMENTS

- Physical quantity = Numerical value × unit
- Homogeneity Principle
  Dimensions of [LHS] = Dimensions of [RHS]
- Mean absolute error

$$\Delta a_{\text{mean}} = \frac{|\Delta a_1| + |\Delta a_2| + ... + |\Delta a_n|}{n}$$

$$\Delta a_{\text{mean}} = \frac{1}{n} \times \sum_{i=1}^{n} |\Delta a_i|$$

- Arithmetic mean $a_{\text{mean}} = \frac{a_1 + a_2 + ... + a_n}{n}$

$$a_{\text{mean}} = \frac{1}{n} \times \sum_{i=1}^{n} a_i$$

- Relative error of fractional error

$$= \frac{\text{mean absolute error}}{\text{mean value}} = \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}}$$

- Percentage error $\delta a = \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}} \times 100\%$
- If in a vernier callipers n VSD coincide with (n – 1) MSD, then vernler constant or its least count is $VC = \left(1 - \frac{n-1}{n}\right)$ (value or 1 MSD) or $\frac{1}{n}$ (value of MSD).
- Least count of screw gauge or spheromenter

$$= \frac{\text{Pitch}}{\text{Number of divisions on circular scale}} \text{ and}$$

$$\text{Pitch} = \frac{\text{Number of divisions moved on linear scale}}{\text{Number of rotations given}}$$

  = Linear distance moved in one rotation.
- Random error = $\sqrt{n}$, where n = number of events or n = number of quantities.
- Radius of curvature using spherometer

$$R = \frac{l^2}{6h} + \frac{h}{2}$$

UNITS AND MEASUREMENTS

- Speed $= \frac{\text{total path length}}{\text{time taken}}$
  - Average speed $= \frac{\text{total distance travelled}}{\text{total time taken}}$

  i.e. $\upsilon_{av} = \frac{S_1 + S_2 + S_3.......}{t_1 + t_2 + t_3.........}$

  - Instantaneous speed

$$= \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

- Velocity $= \frac{\text{displacement}}{\text{time taken}}$
  - Average Velocity $= \frac{\text{total displacement}}{\text{total time taken}}$
- acceleration $a = \frac{\text{change in velocity}}{\text{time taken}}$
  - Average acceleration

$$\vec{a}_{av} = \frac{\Delta \vec{\upsilon}}{\Delta t}$$

  - Instantaneous acceleration

$$\vec{a} = \lim_{\Delta t \to 0} \left(\frac{\Delta \vec{\upsilon}}{\Delta t}\right)$$

- Equation of motion for a uniform accelerated motion
  - $\upsilon = u + at$
  - $s = ut + \frac{1}{2}at^2$
  - $\upsilon^2 - u^2 = 2as$
  - $S_n = u + \frac{a}{2}(2n - 1)$

  Where u is initial velocity, υ is final velocity, a is uniform acceleration, s is distance travelled in tiem t, s_n is distance covered in n^{th} second. These equations are not valid if acceleration is non-uniform.
- Equation of motion for a body under gravity
  - $\upsilon = u + gt$

- $h = ut + \frac{1}{2}gt^2$
- $\upsilon^2 - u^2 = 2gh$
- $h_n = u + \frac{1}{2}g(2n-1)$

- Relative velocity
  - If two bodies are moving along the same line in the same direction with velocities υ_A and υ_B relative to earth, the velocity of B relative to A will be given by $\upsilon_{BA} = \upsilon_B - \upsilon_A$.
- RElative velocity of rain

  tan $\alpha = \frac{\upsilon_m}{\upsilon_r}$

  Where, υ_m = velocity of man υ_r =velocity of rain and α is the angle with the vertical direction at which man should hold umbrella to save himself from the rain.

- Unit vector, $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$

  where, $\hat{a}$ is the unit vector drawn in the direction of $\hat{a}$ and $|\vec{a}|$ is the magnitude of the vecotr.
- Dot or scalar product

  $\vec{a} \cdot \hat{b} = ab \cos\theta$,

  $0 \leq \theta \leq \pi$
- Properties of dot product
  - $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$
  - $\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c}$
  - $m(\vec{a}\cdot\vec{b}) = m\vec{a}\cdot\vec{b} = \vec{a}\cdot(m\vec{b}) = (\vec{a}\cdot\vec{b})m$

    where m is a sclar
  - $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1,$
  - $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{j} = 0$
  - If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and

    $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

    $\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

    $\vec{a}\cdot\vec{a} = a^2a_1^2 + a_2^2 + a_3^3.$

    $\vec{b}\cdot\vec{b} = b^2 = b_1^2 + b_2^2 + b_3^3.$
  - If $\vec{a}\cdot\vec{b} = 0$ and $\vec{a}$ and $\vec{b}$ are not null vectors, then $\vec{a}$ and $\vec{b}$ are perpendicular.
- Cross or vector product

  $\vec{a}\times\vec{b} = ab\sin\theta\hat{n}.$
- Properties of vector product
  - $\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$
  - $\vec{a}\times(\vec{b}+\vec{c}) = \vec{a}\times\vec{b} + \vec{a}\times\vec{c}$
  - $m(\vec{a}\times\vec{b}) = (m\vec{a})\times\vec{b} = \vec{a}\times(m\vec{b}) = (\vec{a}\times\vec{b})m,$
  - $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0, \hat{i}\times\hat{j}$

    $= \hat{k}, \hat{j} = \hat{k} = \hat{i}, \hat{k}\times\hat{i} = \hat{j}$
  - If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

    and $\vec{b} = b_2\hat{i} + b_2\hat{j} + b_3\hat{k}$, then

    $$\vec{a}\times\vec{b}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$
  - $\vec{a}\times\vec{b} =$ than area of a parallelogram with sides $\vec{a}$ and $\vec{b}$.
  - If $\vec{a}\times\vec{b} = 0$ and $\vec{a}$ and $\vec{b}$ are not null vectors, then $\vec{a}$ and $\vec{b}$ are parallel.
- Parallelogram law of vector addition

  $\vec{R} = \vec{a}\times\vec{b}$, than $= \vec{R} = \sqrt{a^2 + b^2 + 2a\cos\theta}$

  and than $\beta = \frac{b\sin\theta}{a + b\cos\theta}$
  - If $\vec{R} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b})$

  than $R = \sqrt{a^2 + b^2 - 2ab\cos\theta}$

  and tan $\beta = \frac{b\sin(180^o - \theta)}{a + b\cos(180^o - \theta)} = \frac{b\sin\theta}{a - b\cos\theta}$

  Where, θ is the angle between $\vec{a}$ and $\vec{b}$.
- Equation of trajectory

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

Where u is initial velocity markes an angle θ with the horizontal.

- Time of flight

$$T = \frac{2u\sin\theta}{g}$$

- Horizontal range

$$R = \frac{u^2\sin 2\theta}{g}$$

  - Range will be maximum.
    if $\theta = 45^\circ$

$$R_{\max} = \frac{u^2}{g}$$

  - If angle of projection is changed from θ to $\theta = (90^\circ - \theta)$ than range

$$R' \frac{u^2\sin 2\theta}{g} = \frac{u^2\sin[2(90^\circ - \theta)}{g}$$

$$= \frac{u^2\sin 2\theta}{g} = R$$

- Maximum Height

$$H = \frac{u^2\sin^2\theta}{2g}$$

  - Height attained by projectile is maximum
    if $\theta = 90^\circ$

$$H_{\max} = \frac{u^2}{2g} = \frac{R_{\max}}{2}$$

  - Here, range of projectile

$$R = \frac{u^2\sin 2\times 90^\circ}{g} = 0$$

  - When the range is maximum, $(\theta = 45^\circ)$

$$H = \frac{u^2\sin^2 45^\circ}{2g} = \frac{u^2}{4g} = \frac{R_{\max}}{4}$$

- Projectile on an inclined plane
  (Motion up the Plane)
  - Time of flight $T = \dfrac{2u\sin(\theta - \beta)}{g\cos\beta}$
  - Range

$$R = \frac{u^2[\sin(2\theta - \beta) - \sin\beta]}{g\cos^2\beta}$$

R will be maximum when $\sin(2\theta - \beta)$ is maximum.
i.e. $\sin(2\theta - \beta) = 1$

$$R_{\max} = \frac{u^2}{g(1+\sin\beta)} \text{ up the plane}$$

- Motion down the plane
  - Time of flight $T = \dfrac{2u\sin(\theta + \beta)}{g\cos\beta}$
  - Range, $R = \dfrac{u^2}{g}\left[\dfrac{\sin(2\theta + \beta) + \sin\beta}{1\sin^2\beta}\right]$

R will be maximum, if $\sin(2\theta + \beta) = 1$

$$R_{\max} = \frac{u^2}{g}\left[\frac{1+\sin\beta}{1-\sin^2\beta}\right] = \frac{u^2}{g(1-\sin\beta)} \quad \text{down}$$

the plane

- At the highest point of a projectile motion given angular projection, the angular momentum of projectile.

$$L = mu\cos\theta \times \frac{u^2\sin^2\theta}{2g}$$

- In case of angular projection, the angle between velocity and acceleration varies from $0^\circ < \theta < 180^\circ$.
- Angular acceleration

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

- When a body moves in a circular path with increasing angular velocity, it has two linear acceleration.
  - Centripetal accleration

$$a_c = \frac{v^2}{r} = e\omega^2 = v\omega = v(2\pi v)^2$$

- Tangential acceleration

  $a_t = r\alpha$

  Resultant acceleration

  $$a = \sqrt{a_c^2 + a_t^2}$$

  $$\tan\beta = \frac{a_t}{a_c}.$$

- Centripeta l force

  $$F = \frac{mv^2}{r}.$$

LAWS OF MOTION

- Linear momentum

  $\vec{p} = m\vec{v}$

  Where, m is mass of a body moving with velocity $\vec{v}$.

- Newton's second law

  Force, $\vec{F} =$ rate of change of linear momentum

  $$= \frac{d\vec{p}}{dt} = m\vec{a}.$$

  Where $\vec{a}$ is acceleration produced in the body.

- Impulse = chanhe in linear momentum

  $$= F \times t = m(v - u)$$

- Equilibrium of concurrent forces :

  $$\vec{F_1} + \vec{F_2} + \vec{F_3} + ...\vec{F_n} = 0$$

- Lamit's theorem :

  $$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

  Where, $\alpha =$ angle between $\vec{F_2}$ and $\vec{F_3}$

  $\beta =$ angle between $\vec{F_3}$ and $\vec{F_2}$

  $\gamma =$ angle between $\vec{F_1}$ and $\vec{F_2}$

- Apparent weight of a man a lift :
  - When the lift is at rest or moving with constant velocity, the apparent weight = *mg*.Thus appare nt weight = true weight.
  - When the lift is accelerating upwards with acceleration a, then apparent weight = $m\ (g + a)$.
  - Thus apparent weight is more than the true weight.
  - When the lift is accelerating downwards with acceleration a, then apparent weight = $m\ (g - a)$

    Thus apparent weight is less than the true weight of man.

- In the cable supporting the lift breaks, the lift falls freely with a = g, then apparent wight

  $$= m(g - g) = 0.$$

- When a person of mass m climbs up a rope with acceleration a, the tension in the rope is $T = m(g + a)$.

- When the person climbs down the rolpe with uniform speed, the te nsion in the rope is $T = mg$.

- Thrust on the rocket $F = -u\left(\frac{dm}{dt}\right)$

  Where, $\frac{dm}{dt}$ is mass of burnt gases escaping per second and u = exhaust speed of the burnt gases.

- Velocity of rocket at any time *t*.

  $$v = u\log_e\left(\frac{m_o}{m}\right)$$

- Accleration of rocket at any instant

  $$a = \frac{\text{upthrust-weight}}{\text{mass}}$$

- Laws of friction :
  - The mag nitude of the force of static fric tion between any two surfaces in contact can have the values

    $f_s \le \mu_s R$ ...(i)

  where the dimensionless constant μ_s is called the coefficient of static friction, R is the magnitude of normal reaction force. The equality in equation.

- holds when the surface are on the verge of slipping i.e., $f_s = (f_s)_{\max} = (f_t) = \mu_s R.$
- The magnitude of the force of kinetic friction acting between two surface is

$$f_k = \mu_k R$$

where μ_k is coefficient of kinetic friction.

- Acceleration of a body down a rough inclined plane, $a = g(\sin\theta - \mu\cos\theta)$

  where, θ is the ngle of inclination and μ is the coefficient of friction.

- Angle of repose

  $\mu = \tan\alpha$

  Where, α is angle of repose

- Work done in moving a body over a rough horizontal surface.

$$W = \mu R \times s = \mu mg \times s$$

Where, R is normal reaction and s is distance moved by body.

- Work done in moving a body up a rough inclined plane.

$$W = (mg\sin\theta + \mu R)s.$$

- Bending cyclist,

  angle of bendiong tan $\tan\theta = \dfrac{v^2}{rg}$

- Circular turning of roads
  - The velocity with which a car can ta ke a cir can take a circular path of radius r without slipping is given by $v_{\max} = \sqrt{\mu_s rg}$
  - The maximum permissible speed to avoid slipping,

$$v_{\max} = \left[\frac{rg(\mu_s + \tan\theta)}{1 - \mu_s \tan\theta}\right]^{1/2}$$

  - $v_0^2 = rg\tan\theta$ or $\tan\theta = \dfrac{v_0^2}{rg}$
  - $\tan\theta = \dfrac{h}{\sqrt{b^2 - h^2}} = \dfrac{v_0^2}{rg}$

- Motion in a vertical circle
  - Tension at any position of angular displacement, (θ) along a vertical circle is given by

$$T = \frac{mv^2}{r} + mg\cos\theta$$

  - At the lowest point of vertical circle, $\theta = 0^{o}$ Tension at the lowest point is given by

$$T_L = \frac{mv_L^2}{r} + mg$$

  - At the highest point of the vertical circle, $\theta = 180^{o}$. Tension at the highest point is given by

$$T_H \frac{mv_H^2}{r} - mg$$

  - Minimum velocity at the highest point,

$$v_H = \sqrt{gr}$$

  - Minimum ve locity at the lowest point for looping the loop, $v_L = \sqrt{5gr}$
  - When the string is horizonta l, $\theta = 90^{o}$, minimum velocity, $v = \sqrt{3gr}$.
  - Height through which a body should fall for looping the vertical loop $h = 5r/2$.

WORK, ENERGY AND POWER

- $W = \vec{F}.\vec{S} = FS\cos\theta$

  Where θ is angle between $\vec{F}$ and $\vec{S}$

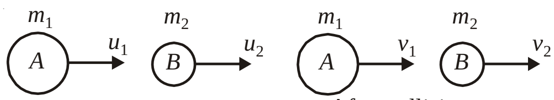
- Work done by a variable force, $W = \int_{x_1}^{xf} F(x)dx$
- Kinetic energy : $K = \dfrac{1}{2}mv^2$.
- Relation between kinetic (K) and linear momentum (p)

$$K = \frac{p^2}{2m} \; or \; p = \sqrt{2mK}$$

Work done by a spring force

$$W = \int \vec{F}_{\text{spring}} . \vec{ds}$$

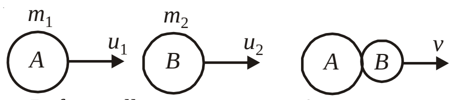
- Work energy theorem : $W = K_f - K_i$
- Elastic potential energy : $U = \frac{1}{2}kx^2$
- Gravitational potential energy : $U = mgh$
- Power, $P = \frac{W_{\text{total}}}{t}$
- Instantaneous power, $P = \frac{dW}{dt} = \vec{F}.\frac{d\vec{s}}{dt} = \vec{F}.\vec{v}$
- Elastic colision in one dimension



$$v_1 = \frac{(m_1 - m_2)u_1}{(m_1 + m_2)} + \frac{2m_2u_2}{m_1 + m_2}$$

$$v_2 = \frac{2m_1u_1}{m_1 + m_2} + \frac{(m_2 - m_1)}{m_1 + m_2}u_2$$

- Perfectly inelastic collision in one dimension



$$v = \frac{m_1u_1 + m_2u_2}{(m_1 + m_2)}$$

- Loss in kinetic energy in elastic in elastic collision is

$$\Delta K = \frac{1}{2}\frac{m_1m_2}{(m_1 + m_2)}(u_1 - u_2)^2$$

- Coefficient of restitution

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

- Kinetic energy lost in inelastic collision is

$$\Delta K = \frac{1}{2}\frac{m_1m_2}{(m_1 + m_2)}(u_1 - u_2)^2(1 - e^2)$$

- A ball falls from a height h, it strikes the ground with a ve locity $u = \sqrt{2gh}$. Let it rebound with a velocity v and rise to a height h_1.

$$e = \frac{v}{u} = \frac{\sqrt{2gh_1}}{\sqrt{2gh}} = \sqrt{\frac{h_1}{h}}$$

or $\sqrt{h_1} = e\sqrt{h}$ or $h_1 = e^2h.$

- A ball dropped from a heigh h and rebounding. The time taken by the ball in rising to height h_1 and coming back is $2\sqrt{\frac{2h_1}{g}} = 2e\sqrt{\frac{2h}{g}}.$

ROTATIONAL MOTION

- The coordinates of centre of mass are given by

$$X_{CM} = \frac{\sum_{i-1}^{N} m_i x_i}{\sum_{i=1}^{N} m_i} = \frac{\sum_{i-1}^{N} m_i x_i}{M}$$

$$Y_{CM} = \frac{\sum_{i-1}^{N} m_i y_i}{\sum_{i=1}^{N} m_i} = \frac{\sum_{i-1}^{N} m_i y_i}{M}$$

$$Z_{CM} = \frac{\sum_{i-1}^{N} m_i Z_i}{\sum_{i=1}^{N} m_i} = \frac{\sum_{i-1}^{N} m_i Z_i}{M}$$

where $M = m_1 + m_2 + m_3 ... m_N$ (total mass of system)

- For a continuous distribution of mass, the coordinates of centre of mass are given by

$$X_{CM} = \frac{1}{M}\int adm; Y_{CM} = \frac{1}{M}\int ydm; Z_{CM}$$

$$= \frac{1}{M}\int zdm$$

- Velocity of centre of mass is given by

$$\vec{v}_{CM} = \frac{\sum_{i=1}^{N} m_i \vec{v_i}}{\sum_{i=1}^{N} m_i} = \frac{\sum_{i=1}^{N} m_i \vec{v_i}}{M}$$

- Acceleration of centre of mass is given by

$$\vec{a}_{CM} = \frac{\sum_{i=1}^{N} m_i \vec{a_i}}{\sum_{i=1}^{N} m_i} = \frac{\sum_{i=1}^{N} m_i \vec{a_i}}{M}$$

- Angular ve locity : $\omega = \frac{d\theta}{dt}$
- Angular acceleration : $\alpha = \frac{d\theta}{dt}$
- Equations of rotational motion
  - $\omega = \omega_0 + \alpha t$
  - $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
  - $\omega^2 - \omega_0^2 = 2\alpha\theta$
- Torque $\vec{\tau} = \vec{r} \times \vec{F}$

  In magnitude $\tau = rF\sin\theta$
- Angular momentum $\vec{L} = \vec{r} \times \vec{p}$

  In magnitude, $L = rp\sin\theta$
- Relationship between torque and angular moment

  i.e., $\vec{\tau}_{ext} = \frac{d\vec{L}}{dt}$
- Moment of inerTIA : $i = \sum_{i=1}^{N} m_i r_i^2$
- Theorem of perpendicular axis : $I_2 = I_x + I_y$

  where, x and y are two perpendicumlar to axes in the plane and z axis is perpendicular to its plane.
- Theorem of parallel axes : $I = I_{CM} + Md^2$

  where, I_{CM} is the moment of ineratia of the body about an axis passing through the centre of mass and d is the perpendicular distance between two parallel axis.

S.No.	Body	Axis of rotation	Moment of inertia (I)	Radius of gyration (K)
1.	Uniform circular ring of mass M and radius R	(i) about an axis passing through its centre and perpendicular to its plane	MR^2	R
		(ii) about a diameter	$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$
		(iii) about a tangent in its own plane	$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$
		(iv) about a tangent perpendicular to its plane	$2MR^2$	$R\sqrt{2}$
2.	Uniform circular disc of mass M and radius R	(i) abojut an axios passing through its centre and perpendicular to its plane	$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$
		(ii) about a diameter	$\frac{1}{4}MR^2$	$\frac{R}{2}$
		(iii) about a tangent in its own plane	$\frac{5}{4}MR^2$	$\sqrt{5}\frac{R}{2}$

		(iv) about a tangent perpendicular to its own plane	$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$
3.	Solid sphere of radius R and mass M	(i) about its diameter	$\frac{2}{5}MR^2$	$\sqrt{\frac{2}{5}}R$
		(ii) about a tangetial axis	$\frac{7}{5}MR^2$	$\sqrt{\frac{7}{5}}R$
4.	Hollow sphere of radius Rand mass M	(i) about its diameter	$\frac{2}{3}MR^2$	$\sqrt{\frac{2}{3}}R$
		(ii) about a tangential axis	$\frac{5}{3}MR^2$	$\sqrt{\frac{5}{3}}R$
5.	Solid cylinder of length l, radius R and mass M	(i) about its own axis	$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$
		(ii) about an axis passing through its centre and perpendicular to its own axis	$M\left[\frac{l^2}{12}+\frac{R^2}{4}\right]$	$\sqrt{\frac{l^2}{12}+\frac{R^2}{4}}$
		(iii) about the diameter of one of the of cylinder	$M\left[\frac{l^2}{3}+\frac{R^2}{4}\right]$	$\sqrt{\frac{l^2}{3}+\frac{R^2}{4}}$
6.	Thin rod of length L	(i) about an axis through its centre and perpendicular to the rod	$\frac{ML^2}{12}$	$\frac{L}{\sqrt{12}}$
		(ii) about an axis through one and perpendicular to the rod.	$\frac{ML^2}{3}$	$\frac{L}{\sqrt{3}}$

- Relation between torque and moment of inertia Tprque $\tau = I\alpha$
  where α is the angular acceleration.
- Relation between angular moment and momemt of inertia, $L = I\omega$
- Kinetic energy of rotatiobnal motion, $K_R = \frac{1}{2}I\omega^2$.
- Kinetic energy of a rolling body = translational kinetic energy (K_T) + rotational kinetic energy (K_R)

$$= \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2\left[1+\frac{K^2}{R^2}\right]$$

- When a body rolls down an inclined plane opf inclination θ without slipping its ve locity at the bottom of incline is give n by $v = \sqrt{\frac{2gh}{1+\frac{K^2}{R^2}}}$

where h is the height of the incline.

- When a body rolls down on an inclined plane without slipping, its acxceleration down the inclined plane is given by $a = \frac{g\sin\theta}{1+\frac{K^2}{R^2}}$.
- When a body roll down on an inclined plane without slipping, time taken by the body to reach the bottom is given by $t = \sqrt{\frac{2l\left(1+\frac{K^2}{R^2}\right)}{g\sin\theta}}$

where *I* is the length of thew inclined plane.

GRAVITATION

- Newton's universal lae of gravitation

$$F = \frac{Gm_1m_2}{r^2}$$

Where , *r* is the separation between masses of objects m_1 and m_2.

- Acceleration due to gravity

$$g = \frac{GM}{R^2}.$$

Where *M* and *R* are the mass and radius of Earth respecxtively.

- Relationship between *g* and *G*

$$g = \frac{GM_e}{R_e^2} = \frac{G\frac{4}{3}\pi R_e^3\rho}{R_e^2} = \frac{4}{3}\pi G R_e\ \rho$$

where M_e is the mass of the earth, R_e is the radius of the earth and ρ is the uniform density of the material of the earth.

- The acceleration due to gravity at height h above *h* above the surface of earth is given by

$$g_h = \frac{GM_e}{(R+h)^2} = g\left(1+\frac{h}{R_e}\right)^{-2} \left(\therefore g = \frac{GM_e}{R_e^2}\right)$$

For $h << R_e$

$$\therefore g_h = g\left(1-\frac{2h}{R_e}\right)$$

- The acceleration due to gravity at a depth d below the surface of earth given by

$$g_d = \frac{GM_e}{R_e^3}(R_e - d) = g\left(\frac{R_e - d}{R_e}\right)$$

$$= g\left(1-\frac{d}{R_e}\right)$$

- At the centre, $d = R_e \ \therefore g_d = 0.$
- Fravitational field intensity

$$\vec{E} = -\frac{Gm}{r^2}$$

Where, *m* is test mass.

- The gravitational field intensity due to spherical shell of radius *R* and *M* at a point distant r from the centre of shell is given as follows :
  - At a point outside the shell *i.e.* $r > R$

$$E = -\frac{GM}{r^2}$$

  - At a point on the surface of the shell *i.e.r* =*R*

$$E = -\frac{GM}{R^2}$$

  - At a point inside the shell *i.e.r.* <*R*, *E* =0
- For solid sphere gravitational field intensity change only at a point inside the sphere *i.e.*, $r < R$.

$$E = -\frac{GMr}{R^3}$$

- Gravitational potential : $V = -\frac{GM}{r}$
- The gravitational potential due to a spherical shell of radius R and mass M at a point diostant r from the centre of the shell is give n as follows :
  - At a point outside the shell *i.e.r.* > *R*

$$V = -\frac{GM}{r}$$

  - At a point on the surface of the shell *i.e.r* =*R*

$$V = -\frac{GM}{R}$$

  - At a point inside the shell *i.e.r.* > *R*

$$V = -\frac{GM}{R}$$

- The gravitational potential due to a solid sphere at a point inside the sphere *i.e.r.* < *R*

$$V = -\frac{GM(3R^2 - r^2)}{2R^3}$$

- Gravitational potential potential energy :

$$U = -\frac{GMm}{r}$$

- Gravitational potential energy of a body of mass m at height h above the surface of the earth is given by

$$U_h = \frac{-GM_e m}{(R_e + h)}$$

- Gravitational potential energy of a body of mass m on the surface of the earth is given by

$$U_s = \frac{-GM_e m}{R_e}$$

- Orbital speed of satellite, when it is revoluing around earth at a height *h* is given by

$$v_0 = \sqrt{\frac{GM_e}{R_e + h}} = R_e\sqrt{\frac{g}{R_e + h}} \quad \left(\text{As } g = \frac{GM_e}{R_e^2}\right)$$

- Time period of a satellite :

$$T = 2\pi\sqrt{\frac{(R_e + h)^3}{GM_e}} = \frac{2\pi}{R_e}\sqrt{\frac{(R_e + h)^3}{g}}$$

- Angular momentum of a satellite

$$L = mv_0 r = mr\sqrt{\frac{GM}{r}} = [m^2 GMr]^{1/2}$$

- Kinetic energy of a satellite,

$$K = \frac{1}{2}mv_0^2 = \frac{1}{2}\frac{GM_e m}{(R_e + h)} = \frac{|U|}{2}$$

- Potential energy of a satellite, $U = \frac{-GM_e m}{R_e + h}$
- Total energy (mechanical) of a satellite

$$E = K + U = -\frac{GM_e m}{2(R_e + h)}$$

- Escape speed : $v_e = \sqrt{\frac{2GM_e}{R_e}}$

PROPERTIES OF SOLIDS

- Stress $= \frac{\text{restoring}}{\text{area}}$
- Longitudianal stress $= \frac{F_N}{A}$
- Volumetric stress $= \frac{F_V}{A}$
- Tangential stress $= \frac{F_T}{A}$
- Longitudinal strain $= \frac{\text{change in length}}{\text{original length}} = \frac{\Delta L}{L}$
- Volumetric strain $= \frac{\text{change in volume}}{\text{original volume}} = \frac{\Delta V}{V}$
- Hooke's law : $= E \times \text{Strain}$ or $\frac{\text{Stress}}{\text{Strain}} = E$
  - Young's modulus, $Y = \frac{\text{normal stress}}{\text{longitudinal strain}}$

$$= \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L} = \frac{FL}{\pi r^2 \Delta L}$$

  - Bulk modulus, $B = \frac{\text{normal stress}}{\text{volumetric strain}}$

$$= \frac{-F/A}{\Delta V/V} = -\frac{PV}{\Delta V}.$$

$-$ve sign shows that volume is decreasing when force id applied.

  - Modulus of rigidity $(\eta) = \frac{\text{tangential stress}}{\text{shearing strain}}$

$$= \frac{F/A}{\theta} = \frac{F}{A\theta}.$$

- In case of a solids and liquids bulk modulus is almost constant.
- In case of a gas, it is process dependent
  - In isothermal process, $K = K_i = P$
  - In adiabatic process $K = K_a = \gamma P$
- Compressiblity $= \frac{1}{\text{Bulk modulus } (B)}$

when pressure is applied on a substance, its volume decreses while mass remains constant. Hence , its density will increases,

$$\rho' = \frac{\rho}{1 - \Delta P / B} \text{ or } \rho' = \rho\left(1 + \frac{\Delta P}{B}\right) \text{ if } \frac{\Delta P}{B} << 1$$

- Poisson's ratio

$$(P): \frac{\text{lateral strain}}{\text{longitudinal strain}} = -\frac{\Delta e / r}{\Delta L / L}$$

- Relations among eleastic constants $(Y, B, \eta \text{ and } \sigma)$
  - $Y = 3B(1 - 2\sigma)$
  - $Y = 2\eta(1 + \sigma)$,
  - $\sigma = \frac{3B - 2\eta}{2\eta + 6B}$,
  - $\frac{9}{Y} = \frac{1}{B} + \frac{3}{\eta}$
- Breaking force = Breaking stress ×Area of cross section of the wire.
- Every wire is like a spring whose force constant is equal to

$$K = \frac{YA}{L} \; or \; K \propto \frac{1}{L}$$

- Work done in a stretched wire,

$$W = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{colume}$$

$$= \frac{1}{2}\frac{F}{A} \times \frac{\Delta L}{L} \times AL = \frac{1}{2} F \times \Delta L$$

$$= \frac{1}{2} \times \text{load} \times \text{elongation}$$

- Elastic potential energy per unit volume of a stretched wire,

$$u = \frac{1}{2} \times \text{stress} \times \text{strain} = \frac{1}{2} \times Y \times (\text{strain})$$

In case of elongation by its own weight, $F(= Mg)$ will act at centre of gravity of the wire, so that length of wire which is stretched is (L/2).

$$\therefore \; \Delta L = \frac{Mg(L/2)}{AY} = \frac{MgL}{2AY} = \frac{\rho g L^2}{2Y}$$

$$[\because M = \rho AL]$$

- In case kof twisting of a cylinder (or wire) of lenght L and radius r, elastic restoring couple per unit twist is given by

$$C = \frac{\pi \eta r^4}{2L}$$

where η is modulus of rigidity of the material of wire.

- Interatomic force constant (k)

$$k = \frac{\text{interatomic force}}{\text{change in interatomic distance}} = \frac{F_0}{\Delta r} = Yr_0$$

- Depression of a beam loaded at the middle by a load W and supported at the ends

$$\delta = \frac{WL^3}{48YI_g}$$

- Depression of a cantilever at a free end

$$\delta = \frac{WL^3}{3YI_g}$$

PROPERTIES OF FLUIDS

- Density, $\rho = \frac{m\,(\text{mass})}{V(\text{volume})}$
- Relative density $= \frac{\text{density of substance}}{\text{density of water at } 4^{\circ}\text{C}}$
- Pressure $P = \frac{\text{thrust } (F)}{\text{area } (A)} = \frac{F}{A}$
- For a point at a depth h below the surface of a liquid of density ρ, hydrostatic pressure P is given by $P = P_0 + h\rho g$

  where P_0 represents the atmospheric pressure.
- When a body of density ρ_B (which may be different from the density of material of body) and volume V is completely immersed in a liwuid of debnsity σ , following two forces act on the body :
  - weight of body $W = V\rho_B g$ acting verti cally downwards through the centre of gravity. Buoyant force or upward thrust $W' = V\sigma g$ equal to weight of the liquid dis-

placed, acting vertically upwards through the centre of buoyancy.

- Depending ukpon relative magnitudes of above two forces, following three cases are possible :
  - Thedensity of body is greater than that of liquid $(i.e., \rho_B > \sigma)$. In this situation as weight wil be more than ukpthrust, the body will sink.
  - The density of body is equal to the density of liquid $(i.e., \rho_B > \sigma)$. In this situation W =W' so the body wil move upwards and in equilibrium will float partially immersed in the liquid such that

    $W = V_{in}\sigma g$ or $V\rho_B g$ or $V\rho_B = V_{in}\sigma$

- Equation of continuity $A_1 v_1 = A_2 v_2$
- Surface tension, $S = \frac{\text{Force}}{\text{Length}} = \frac{F}{L}$
- Work done oin forming a liquid drop of radius r, surface tension S is,

$$W = 2 \times 4\pi r^2 S = 8\pi r^2 S.$$

- Work done in increasing the radius of a liquid drop from r_1 to r_2 is

$$W = 4\pi S\left(r_2^2 - r_1^2\right).$$

- Work done in increasiong the radius of a soap bubble from r_1 to r_2 is

$$W = 8\pi S\left(r_2^2 - r_1^2\right).$$

- When n number of smaller drops of a liquid, each of radius r, surface tenion S are combined to form a bigger drop of radius R , then

$$R = n^{1/3} r$$

- The surface area of bigger drop $= 4\pi R^2 = 4\pi n^{2/3} r^2$. It is less than the area of n smaller drops.
- Work done in breaking a l;iquid drop of radius R into n equal small drops

$$W = 4\pi R^2 (n^{1/3} - 1)S$$

where S is the surface tension.

- Excess pressure inside a liquid drop is given by

$$P = \frac{2S}{r}.$$

- Excess pressure inside a soap bubble is give n by

$$P = \frac{4S}{r}.$$

- Excess pressure inside an air bubble in a liquid is given by

$$P = \frac{2S}{r}.$$

- When an air bubble of radius r is at depth h below the free surface of liquid of density ρ and surface te nsion S, then the excess pressure inside the bubble,

$$P = \frac{2S}{r} + h\rho g.$$

- If r_1 and r_2 are the radiii of curved liquid surface, then excess pressur inside the liquid surface is given by

$$P = S\left(\frac{1}{r_1} + \frac{1}{r_2}\right).$$

- When two soap bubbles of radii r_1 and r_2 coalesce to form a new soap bubble of radius r, under kisothermal conditions then $r = \sqrt{r_1^2 + r_2^2}$.
- When two soap bubbles of radii r_1 and r_2 are in conta ct with each other and r is the radius of the interface, then $r = \frac{r_1 r_2}{r_2 - r_1}$.
- The total pressure inside an air bubble of radius r at a depth h below the surface of liquid of density ρ is

$$P = P_0 + h\rho g + \frac{2S}{r}$$

- The rise or fall in a capillary tube is given by

$$h = \frac{2S\cos\theta}{e\rho g} = \frac{2S}{R\rho g} \qquad \left(\therefore \cos\theta = \frac{r}{R}\right)$$

where θ is theangle of contact.

- According to Newton viscous force (F) of a liquid between two layers is given by

$$F = -\eta A \frac{dv}{dx}$$

where η = coefficient of ciscosity of the liquid

- Poiseuille's equation : $Q = \frac{\pi P r^4}{8\eta l} = \frac{P}{R}$

$R = \frac{8\eta l}{\pi r^4}$ is called liquid resistance.

- Stroke's law : $F = 6\pi\eta rv.$
- Terminal velocity : $v_T = \frac{2r^2(\rho - \sigma)g}{9\eta}.$
- Critical velocity : $v_c = \frac{K\eta}{\rho r}$
- Reynold nu mber : $v_c = \frac{N_R \eta}{\rho D}$ or $N_R = \frac{\rho D v_c}{\eta}$
- Bernoulli's theorem :

$$P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$$

O $P_1 + \rho gh_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho gh_2 + \frac{1}{2}\rho v_2^2$

- Velocity of efflux $v = \sqrt{2gh}$
- Time after while liquid strikes the horiontal syrface

O $t = \sqrt{\frac{2(H-h)}{g}}$

O Range $= R = vt = \sqrt{2gh} \times 2\sqrt{h(Hl-h)}$

O $R_{\max} = H \text{ at } h = \frac{H}{2}$

O If the hole is at the bottom of the tank, time t ta ken by the tank to emptied.

$$t = \frac{A}{a}\sqrt{2H/g}$$

where a is the are of the hole.

THERMAL, PROPERTIES OF MATTER

- Relationship between temperature scales :

$$\frac{T_C - 0}{100} = \frac{T_F - 32}{180} = \frac{T_R - 0}{80}$$

$$= \frac{T_{Ra} - 460}{212} = \frac{T_K - 273.15}{100}$$

- Coefficient of limnear expansiion of a solid,

$$\alpha = \frac{\text{increase in area}}{\text{original length} \times \text{rise in temperature}}$$

$$= \frac{\Delta L}{L \Delta T}$$

- Coefficient of are expansion of a solid,

$$\beta = \frac{\text{increase in area}}{\text{original area} \times \text{rise in temperature}}$$

$$= \frac{\Delta A}{V \Delta T}$$

- Coefficint of volume expansion of a solid,

$$\gamma = \frac{\text{increase in volume}}{\text{original are} \times \text{rise in temperature}} = \frac{\Delta V}{V \Delta T}$$

- Relation between α, β and γ

$$\alpha = \frac{\beta}{2} = \frac{\gamma}{3}.$$

- The specific heat of a su bstance is given by

$$s = \frac{1}{m}\frac{\Delta Q}{\Delta T}$$

- The molar specific heat of a substance is given by

$$C = \frac{1}{\mu}\frac{\Delta Q}{\Delta T}$$

- Thermal capacity, $S = s \times m$
- The latent heat of a substance's given by

$$L = \frac{Q}{m}$$

- Principal of calorimetry :
  Heat lost by one body = Heat ga ined by the other.
- When a bar of length L and uniform are of cross section A with its ends maintaned at temperatures T_1 and T_2, the rate of flow of heat (or heat current) H is given by

$$H = \frac{KA(T_1 - T_2)}{L}$$

- Thermal resistance of the bar, $R_H = \frac{1}{KA}$

- Stefan Boltzmann low : $E = \sigma T^4$
- If the body is not a perfectly b lack body, then $E = \sigma T^4$
- The energy radiated persecond by a body of area $A = eA\sigma T^4$
- Newto n's law of cooling : $\frac{dQ}{dt} = -k(T - T_s)$
- Wien's displacement law : $\lambda_m T$ = constant
- Temperature of sun is given by $T = \left(\frac{R^2 S}{R_s^2 \sigma}\right)^{1/4}$

THERMODYNAMICS

- The work done by a gas is $W = \int dW = \int_{Vi}^{V_f} PdV$

  Where V_i and V_f are the i ntial and final volume of the gas.
- First law of thermodynamics :

  $\Delta Q = \Delta U + \Delta W$
- Equation of istherma process,

  $$W = \mu RT \ln\left(\frac{V_f}{V_i}\right); W = mRT \ln\left(\frac{P_i}{P_f}\right)$$
- Equation of adiabatic process, PV $\gamma =$ constant where $\gamma = C_p / V_v$.
- Work done during adiabatic process,

  $$W = \frac{(P_i V_i - P_f V_f)}{(\gamma - 1)}; W = \frac{\mu R(T_i - T_f)}{\gamma - 1}$$
- Equation of isobaric process $\frac{V}{T}$ = constant.

  O Work done during isobaric process,

  $$W = P(V_f - V_i) = \mu R(T_f - T_i).$$

  Efficient of a heat engine,

  $$\eta = \frac{work\ done}{heat\ absorbed} = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$
- The coefficient of performance of refrigrator,

  $$\beta = \frac{\text{heat extracted from the resevoir at atemperature T}_2}{\text{work done to transfer the heat}}$$

  $$= Q_2 = \frac{Q_2}{Q_1 - Q_2}$$
- The efficiency of a carnot engine is given,

  $$\eta = 1 - \frac{T_2}{T_1}$$

KineticTheory of Gases

- Equation of an ideal gas : $PV = \mu RT = kgNT$

  Boltzmann constant

  $$k_B = \frac{R}{N_A}$$

  N_A is the Avogadro's number.

  Here, $\mu = \frac{m}{M} = \frac{N}{N_A}$

  Where, m isthe mass of the gas containing N molecules, M is the molar mass
- Equation of a real gas :

  $$\left(P + \frac{\mu^2 a}{V^2}\right)(V - \mu b) = \mu RT$$

  where, a and b are Va n der waals constants
- Critical temperature : $T_C = \frac{8a}{27Rb}$
- Critical te mperature : $P_C = \frac{a}{27b^2}$
- Critical volume : $V_C = 3b$
- According to kinetic theory of an ideal gas pressure exerated by an ideal gas given by

  $$P = \frac{1}{3} mn \overline{v^2}$$
- Root mean squre speed,

  $$V_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3k_B T}{m}}$$
- Average speed , $\bar{v} = \sqrt{\frac{8RT}{M}} = \sqrt{\frac{2k_B T}{m}}$
- Most probable speed,

  $$v_{mp} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2k_B T}{m}}.$$

- $v_{rms} > \bar{v} > v_{mp}$
- Average translational kinetic energy of a gas molecule is $E = \frac{3}{2} k_B T$
- The molar specific heats are given by
  - O C_V (rigid diatomic) $= \frac{5}{2} R$
  - O C_P (rigid diatomic) $= \frac{7}{2} R$
  - O γ (rigid diatomic) $= \frac{7}{5}$
- The mean free path, $\lambda = \frac{1}{\sqrt{2} n \pi d^2}$

OSCILLATIONS

- Angular frequency $\omega = 2\pi v = \frac{2\pi}{T}$
- Velocity of a particle in S.H.M. is given by $v = \omega\sqrt{A^2 - x^2}$
- Acceleration of a particle in S.H.M. is given by $a = -\omega^2 x$
- The kinetic energy of a paraticle in S.H.M. is given by $K = \frac{1}{2} m\omega^2 (A^2 - x^2)$
- The potential energy of a particle in S.H.M. is given by, $= \frac{1}{2} m\omega^2 A^2 \sin^2(\omega t + \phi)$
- Total energy of a particle in S.H.M. is given by $E = \frac{1}{2} m\omega^2 A^2$
- Spring pendulum $T = 2\pi \sqrt{\frac{m}{k}}$
- The time period of a simple pendulum is given by $T = 2\pi\sqrt{L/g}$.
- If the length of a simple pend ulum is comparable with the radius of earth (R_e), thewn time period T is given by

$$T = 2\pi \sqrt{\frac{1}{g\left(\frac{1}{L} + \frac{1}{R_e}\right)}}$$

- If a simple pendulu m is suspended in a lift and lift is accelerting downwards with an acceleration a, then its time period is given by

$$T = 2\pi \sqrt{\frac{L}{g - a}}$$

- For upwards motion, $T = 2\pi \sqrt{\frac{L}{g = a}}$
- For upwards or downwards with constant velocity $v, T = 2\pi \sqrt{\frac{L}{g}}$
- If a simple pendulum is suspended in a lift and lift is freely falling with acceleration g, then its time period is given by $T = 2\pi \sqrt{\frac{L}{g - g}} = \infty$
- If a simple pendulum is suspended in carriage which is acclerating horizontally with an acceleration a, then its time period is given by

$$T = 2\pi \sqrt{\frac{L}{\left(\sqrt{g^2 + a^2}\right)}}$$

- If a simple pendulum is suspended from the roof of a trollel which is moving down an inclined plane of inclination θ, then the time period is given by

$$T = 2\pi \sqrt{\frac{L}{g \cos\theta}}$$

- If a simple pendulum whose bob is of density ρ oscillates in a non-visocous liqid of density $\sigma (\sigma < \rho)$,

$$T = 2\pi \sqrt{\frac{L}{\left(1 - \frac{\sigma}{\rho}\right) g}}$$

❒ Torsional pendulum :

$$T = 2\pi\sqrt{\frac{I}{C}}$$

where I is the moment of inertia of the disc about the suspension wire as axis of rotation and C is the restoring torque per unit twit.

$$C = \frac{\pi\eta r^4}{2L}$$

where r is the radius, L is the length and η is the modlus of rigidity of a wire resopectively.

❒ The time period of oscillation of a liquid in U-tube, is given by

$$T = 2\pi\sqrt{\frac{L}{2g} = 2\pi\sqrt{\frac{h}{g}}}$$

where L= total length of liwuid column in a U-tube, h = height of liquid column in each limb Also $h = L/2$

❒ The time period of oscillation of floating cylinder in a liquid is given by $T = 2\pi\sqrt{\frac{m}{A\sigma g}}$

where m is the mass of a cylinder, A is the are of cross section of a cylinder, σ is the density of a liquid

or $T = 2\pi\sqrt{\frac{h\rho}{\sigma g} = 2\pi\sqrt{\frac{h'}{g}}}$

where h is height of cylinder of density ρ and σ is the density of a liquid in which cylinder is floating, h' is the of the cylinder inside the liquid.

❒ Time period of LC oscillations of a circuit containing capacitance C and inductance L is given by

$$T = 2\pi\sqrt{LC}$$

❒ If a wire of length L, are of cross-section A, young's modulus Y is streched by suspending a mass m, then the mass can oscillate with time period

$$T = 2\pi\sqrt{\frac{mL}{YA}}$$

❒ If gas is enclosed in a cylinder of volume V fitted with piston of cross section area A mass M and the piston is slightly depressed and released, the piston can oscillate with a time period

$$T = 2\pi\sqrt{\frac{MV}{BA^2}}$$

WAVES

❒ Speed, frequency and wavelength relation

$$v = \lambda\upsilon$$

❒ Intensity of a wave : $I = 2\pi^2\upsilon^2A^2\rho v$

where υ is the frequency, A is the amplitude, v is the velocity of the wave, ρ is the density of the medium.

❒ Wnergy density, $v = \frac{\omega}{k}$

❒ Particle velocity,

$$v_{\text{particle}} = \frac{dy}{dt} = \omega A\cos(kx - \phi) = -\left(\frac{\omega}{k}\right)\frac{dy}{dx}$$

❒ Particle acceleration, $a = \frac{d^2y}{dt^2} = -\omega^2 y$

❒ Relationship between phase difference, path difference and toime difference

Phase disserence $= \frac{2\pi}{\lambda} \times$ path difference

Phase difference $= \frac{2\pi}{T} \times$ time difference

❒ Speed of a transverse waves on a streched string is given by $v = \sqrt{\frac{T}{\mu}}$

where T is the tension in the string, μ is the mass per unit length of the string callel linear density.

❒ Speed of a transverse wave in a solid is given by

$$v = \sqrt{\frac{\eta}{\rho}}$$

where η is the modulus of rigidity, ρ is the density of a solid.

- Speed of a longitudinal wave in a meduim is given by $v=\sqrt{\frac{E}{\rho}}$

  where E is the modulus of eleasticity and ρ is the density of the medium.

- Speed of a longitudinal wave in a metallic bar is given $v=\sqrt{\frac{Y}{\rho}}$

  where Y is the Young's modulus and ρ is density of a fluld.

- Newon's formula : $v=\sqrt{\frac{P}{\rho}}$

- Speed of sound in a gas, $v=\sqrt{\frac{\gamma}{\rho}}v_{\text{rms}}$

  - Effect of temperature : $v_t = v_0\left[1+\frac{t}{546}\right]$

    where v_0 is the speed of sound in the gas at 0^oC.

  - Effect of pressure : The speed of sound in a gas is given by

    $$v=\sqrt{\frac{\gamma P}{\rho}}=\sqrt{\frac{\gamma RT}{M}}$$

    Speed of sound in gas, provided tempera ture remains constant.

  - Effect of humidity : With increase in hu midity, density of air decreases

    $$v=\sqrt{\frac{\gamma P}{\rho}}$$

- Vibrations in a stretched stretched string of length L fixed at both ends.

  - Fundamental frequency

    $$\upsilon_1=\frac{v}{\lambda_1}=\frac{v}{2L}=\frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

  - For the nth mode, $\lambda_n = 2L/n$

    Frequency of n^{th} mode

    $\upsilon_n=\frac{v}{\lambda_n}=\frac{nv}{2L}=nv_1=\frac{n}{32L}\sqrt{\frac{T}{\mu}}$ where $n =$ 1,2,3...

    $$\upsilon_p=\frac{p}{2L}\sqrt{\frac{T}{\mu}},$$

  where p = number of loops.

- Vibrations of a closed origan piple

  - For n^{th} mode, $\lambda_n=\frac{4L}{(2n-1)}$

- Frequency, $\upsilon_1=\frac{v}{\lambda_n}=\frac{v(2n-1)}{4L}=(2n-1)v_1$

  $$\upsilon_1=\frac{v}{\lambda_1}=\frac{v}{4L}$$

- Due to the end correction the fundamental frequency of a closed organ pipe is given by

  $$\upsilon_C=\frac{v}{4[L+e]}=\frac{v}{4[L+0.6r]}$$

- Due to the end correction, the fundamental frequency of an open pipe is given by

  $$\upsilon_O=\frac{v}{2[L+2e]}=\frac{v}{2[L+1.2r]}$$

- Speed of sound in air at room temprature using resonance tube is given by

  $$v=2\upsilon(L_2-L_1)$$

- Beat frequency = no. of beats/sec = $(\upsilon_1-\upsilon_2)$ =difference in frequencies.

- Tuning fork is a source of sound of single frequency and frequency of a tuning fork of arm length L and thickness d in the direction of vibration is given by

  $$\upsilon=\left[\frac{d}{L^2}\right]v=\frac{d}{L^2}\sqrt{\frac{Y}{\rho}} \qquad \left[\text{since } v=\sqrt{\frac{Y}{\rho}}\right]$$

- According to Doppler's effect the apparent frequency heard by the observer is given by

$$\upsilon' = \upsilon\left[\frac{v \pm v_o}{v \mp v_s}\right]$$

where v_s, v_o and v are the speed of source, when, observer and sound relative to air.

The upper sign on v_o (or v_o) is used when source (observer) moves towards the observer (source) while lower sign is used when it moves away.

❐ If the wind blow with speed v_w in the direction of sound, v is replaced by $v + v_w$ in the above equation. If the wind blow with speed v_w in a direction opposite to that of sound, v is replaced by $v - v_w$ in the above equation.

❐ A practical and small unit of loudness of sound is decibel (dB). 1 decibel =1/10 bel.

❐ In decibe l the loudness of a sound of intensity I is given by $L = 10\log_{10}\left(\frac{I}{I_C}\right)$.

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